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A New angle on excess consumption volatility in emerging countries: Does house price matter?

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ABSTRACT

This paper focuses on house prices in order to explain *excess consumption volatility*, which is one of the stylized facts of the business cycle observed in emerging countries. The relative consumption volatility compared to output volatility is higher in emerging countries than in rich countries. Because house prices in emerging countries are more volatile than in advanced countries, this paper suggests a new angle focusing on relationship between house price fluctuation and excess consumption volatility. To begin, I construct a cross-country dataset, and suggest new evidence that house prices are more correlated with consumption in the case of emerging countries compared to rich countries while the correlation between house prices and output are similar between two country groups. Then, I build business cycle models given exogenous house price and output processes in order to explain excess consumption volatility puzzle. The results of the models suggest that high house price volatility in emerging countries causes their excess consumption volatility and that the mechanisms behind it are housing collateral effects and rental price pass-through. The former is related to the borrowing capacity affected by house price changes, and the latter is related to the link between house prices and housing consumption volatility.
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1. Introduction

The motivations for this paper are born of the stylized facts documented in Uribe and Schmitt-Grohé (2017) and an empirical finding suggested by Cesa-Bianchi et al. (2015). The former is that relative consumption volatility compared to output volatility is higher in emerging countries than in developed countries, and the latter is that house prices in emerging economies are more volatile than those in advanced countries. Based on these two empirical facts, we can postulate a hypothesis that the excess consumption volatility in emerging economies is highly related to greater house price volatility, considering a strong correlation between the aggregate housing price and aggregate consumption investigated in the literature (for instance, Campbell and Cocco, 2007, Case et al., 2005, among them). However, the papers investigating the connection between house prices and consumption do not approach the relationship from the perspective of the business cycle, especially for emerging countries. As a result, this paper attempts to answer the following question: Can we explain the excess volatility of consumption in emerging countries by their relatively high housing price volatility?

Several approaches explain excess consumption volatility in emerging countries, which I classify into three groups: the first group highlights nonstationary output shock as in Aguiar and Gopinath (2007); the second highlights interest rate

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shocks, as in Neumeyer and Perri (2005) and Garcia-Cicco et al. (2010); and the last group involves a more recent paper that argues that volatile durable good consumption is a driving force of excess consumption volatility (Alvarez-Parra et al., 2013). With the purpose of contributing to the literature, this paper explores a new approach that focuses on the channel from house prices to consumption. Specifically, the main goal is to explain the observed excess consumption volatility in emerging economies using higher housing price volatility and examine a possible mechanism behind it.

First, I construct a cross-country dataset containing key macroeconomic variables of interest, including house prices. The main motivational facts — excess consumption volatility and higher house price volatility in emerging countries — are observed in the dataset, and more importantly, there is a higher correlation between house prices and consumption in the case of emerging countries than in rich countries. This higher correlation implies that we can take a new angle focusing on the relationship between housing price volatility and excess consumption volatility in emerging countries because the correlations between housing prices and the output of two country groups are not significantly different. In addition, disaggregated consumption for data-available countries shows interesting features. First, housing consumption is far more volatile than non-housing consumption in emerging countries, while it is less volatile than non-housing consumption in advanced countries. Second, despite considering higher housing consumption volatility, there is still excess volatility in non-housing consumption in emerging countries. Based on the above findings, this paper derives the modeling intuition regarding the mechanisms behind the relationship between housing price volatility and excess consumption volatility in emerging countries. One suggested mechanism is a collateral effect that accounts for excess non-housing consumption volatility, and the other is a rental price pass-through, which explains the link between house prices and total consumption fluctuation through housing consumption.

I then build business cycle models for an emerging small open economy that incorporates disaggregated consumption of households. Specifically, the model explicitly accommodates a housing rental market by a two-agent setting of homeowners and renters. The housing rental price is endogenously determined by transactions between two agents. However, considering the theoretical result of highly volatile rental prices, this paper introduces a controlling device for stable rental prices, which is rental price rigidity. The framework also features a housing collateral constraint and an external debt elastic interest rate. In particular, related to financial friction of borrowing constraint, I suggest two versions of the theoretical model based on the two possible interpretations of the probability of binding constraint; first, in terms of the time dimension, the constraint would be sometimes binding and sometimes unbinding; and second, in view of cross-sectional dimension, some proportion of agents faces a binding constraint and other does not face. Hence, this paper presents two models: two-agent model of homeowners with occasionally binding borrowing constraint and renters, and three-agent model of unconstrained homeowners, constrained homeowners with binding constraint and renters.

To determine the impact of highly volatile house prices, the models assume that the house price is an exogenously-given stochastic process and that housing supply is perfectly elastic. As a result, homeowners are able to purchase as many houses as possible at a given exogenous house price. These assumptions allow the model to concentrate on the mechanism between the house price and aggregate consumption, because we shut down the effect of housing supply and other endogenous sources of housing price determination. The result shows that the house price matters considerably compared to endowment shock in explaining excess consumption volatility; the more volatile the house price is, the more volatile total consumption is in the economy. Additionally, I verify the suggested key mechanisms. First, the collateral effect channel causes the economy to transmit housing price changes into consumption fluctuations, especially, into non-housing consumption fluctuations. Specifically, the housing collateral effect is strong in the case of emerging countries, which this paper supports by implementing comparison analysis with a counterfactual advanced economy. Second, rental price pass-through also works in transmission; that is, the relative price of housing consumption strongly responds to house price fluctuations in the same direction. Hence, the model generates higher housing consumption volatility and further higher total consumption volatility corresponding to higher house price volatility in emerging economies.

The rest of this paper is organized as follows. In the next section, I construct the cross-country dataset and document several stylized facts regarding cyclical fluctuations in house prices and consumption in emerging countries. In Section 3, I present the models. Section 4 describes the calibration and presents the results, and Section 5 performs counterfactual analysis. Finally, Section 6 concludes.

2. Empirical Facts from Data

2.1. A Cross-Country Dataset for Key Indicators

I construct a cross-country dataset containing key macroeconomic variables of interest, including house prices. First, for the house price data, I aim to gather quarterly data as long as possible to avoid short sample bias.[1] The other indicators of interest are each country's output and consumption; the source of both of these indicators is the OECD national account database. Since the quarterly national account data in OECD statistics begin from 1950 Q1, the sample period for this paper's dataset depends on the availability of house price data in the BIS database. The resulting dataset is an unbalanced panel of 54 quarterly time series—house price (HP_t), GDP (Y_t), and consumption (C_t) for 18 countries—with varying coverage from 1970 Q1 to 2016

[1] Please refer to the appendix for the detailed information and discussion related to house price dataset construction.

Q3. The minimum coverage is from 1975 Q1 to 2016 Q3 (167 observations) for South Korea. The sample countries included in the dataset are categorized as either emerging countries or rich countries according to the classification of Uribe and Schmitt-Grohé (2017): 4 emerging countries (New Zealand, South Africa, South Korea, and Spain) and 14 rich countries (Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States).[2] It is notable that there are few emerging countries that have sufficiently long housing price data, as Cesa-Bianchi et al. (2015) mention that collecting housing price data on emerging economies is difficult given the issue of data availability. Although I also initially considered 12 emerging countries, that is, 8 more countries other than New Zealand, Spain, South Korea, and South Africa; these were Brazil, Colombia, Greece, Israel, Malaysia, Peru, Portugal, and Thailand, I decided to drop those countries due to the short sample period (the longest series is for Portugal with 115 observations and the shortest series is for Brazil with 63 observations).[3]

The raw data of HP_t, Y_t, and C_t are typically in nominal terms. After deflating each series by a GDP deflator, I implement seasonal adjustment to only house price series by X-12-ARIMA because HP_t is nonseasonally adjusted data, whereas the raw data of Y_t and C_t are already seasonally adjusted. I then conduct detrending by an HP filter, which implies that the raw series for housing price, GDP and consumption are assumed to be nonstationary. In other words, I eliminate the trend component of each series by inducing stationarity using an HP filter (with $\lambda = 1,600$).[4] Before detrending, in the case of Y_t and C_t, they are divided by population to obtain a per-capita measure. The quarterly population is for all ages and both sexes, which is linearly interpolated from annual population data of the UN Population Division. Next, for the purpose of characterizing average business cycle facts for each country group, I compute second moments of interest for each individual country in the sample and then take an unweighted average across the countries within each respective group.

2.2. Business Cycle Facts: Emerging Countries vs. Rich Countries

In Table 1, I report the particulars of the individual sample country's business cycle as well as the weighted averages by country group. There are three stylized facts that we can observe in Table 1. First, housing price volatility is much larger in emerging countries than in rich countries, which was suggested by Cesa-Bianchi et al. (2015). Second, output is also more volatile in emerging countries, but its magnitude of volatility is lower than the case of house prices. In the case of output, the emerging country group is 1.2 times as volatile as the rich country group, but, in the case of house prices, the former group is 1.4 times as volatile as the latter. Last, emerging countries present higher relative volatility of consumption, or less consumption smoothing. While σ_C/σ_Y for rich countries is 0.88, it is 1.39 in the case of emerging countries. Although we expect lower consumption volatility than income volatility based on the consumption smoothing behavior of households, the expectation holds only for rich country group. The higher relative consumption volatility observed in emerging country group, in particular, σ_C/σ_Y substantially greater than one, is known as *excess consumption volatility*, which is declared in several influential papers including Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010), and Alvarez-Parra et al. (2013).

Table 1 also presents the correlation between key variables: $\rho_{C,Y}$, $\rho_{HP,Y}$, and $\rho_{HP,C}$. The correlation between consumption and output, $\rho_{C,Y}$, is not significantly different between emerging countries and rich countries (0.61 for emerging, 0.68 for rich countries). More importantly, we can also observe a similar correlation between house prices and output, $\rho_{HP,Y}$, between two country groups (0.52 for emerging countries, 0.46 for rich countries), while the correlation between house prices and consumption, $\rho_{HP,C}$, is approximately 1.4 times higher in the emerging country group than in the rich country group (0.54 for emerging countries, 0.39 for rich countries).[5] Note that the similar $\rho_{HP,Y}$ between emerging and rich countries can be interpreted as the control of the effect of output on house prices when we compare two country groups' business cycles. On the other hand, emerging countries present higher $\rho_{HP,C}$ than rich countries, which implies that aggregate consumption is more related to house prices than output in emerging countries. Therefore, the evidence of a higher $\rho_{HP,C}$ for emerging countries implies that higher house price volatility can be a root of the excessive response of consumption for emerging countries. This hypothesis is more valid when we consider the fact that emerging countries' $\rho_{C,Y}$ is similar to rich countries'. As a result, I declare the above as evidence "*Consumption is more correlated with house prices in emerging countries than in rich countries, while the correlation between output and house prices for emerging countries is similar to that of rich countries on average*".

Next, I extend the cross-country dataset to include the disaggregated consumption data series: housing consumption (HC_t) and non-housing consumption (NHC_t). This is because this paper studies the excessive response of household consumption to house price fluctuations, so we need to examine the consumption related to housing and the consumption

[2] Uribe and Schmitt-Grohé (2017) define the country group with the average PPP-converted GDP per capita: poor countries (less than $3,000), emerging countries (between $3,000 and $25,000), and rich countries (above $25,000).

[3] The business cycle facts that I show in this paper are robust for the dataset of 14 emerging economies. Additional details are available from the author upon request.

[4] I check alternative detrending methodologies such as a quadratic filter and a first-differencing filter. Detrending by the log-quadratic filter and the log-difference filter shows similar results to what I earn from the HP filter with a constant 1,600, so I can confirm robustness. Please refer to the appendix.

[5] Between emerging and rich countries, the averages of $\rho_{C,Y}$ and $\rho_{HP,Y}$ are not significantly different, whereas $\rho_{HP,C}$ is significantly different at 10% significance level (p-value is 0.07). Also, I note that the population-weighted (as of 2010) average for each country group yields the same implication for $\rho_{HP,Y}$ and $\rho_{HP,C}$ ($\rho_{HP,Y}$ for emerging country group is 0.51 and that for rich country group is 0.51, and $\rho_{HP,C}$ for emerging country group is 0.49 and that for rich country group is 0.43). In the case of $\rho_{C,Y}$, the correlation is weaker in emerging countries compared to rich countries (0.66 for emerging countries, 0.79 for rich countries).

Table 1
Business Cycles in Emerging and Rich Countries.

	σ_{HP}	σ_Y	σ_C/σ_Y	$\rho_{C,Y}$	$\rho_{HP,Y}$	$\rho_{HP,C}$
New Zealand	6.43	1.76	1.69	0.41	0.54	0.70
South Africa	5.71	1.61	1.60	0.53	0.36	0.64
South Korea	5.16	2.41	1.19	0.70	0.57	0.38
Spain	4.93	1.33	1.06	0.79	0.60	0.44
Emerging Countries	**5.56**	**1.78**	**1.39**	**0.61**	**0.52**	**0.54**
Australia	3.78	1.16	0.94	0.19	0.34	0.12
Belgium	2.80	1.15	0.89	0.75	0.41	0.19
Canada	3.59	1.40	0.79	0.49	0.35	0.22
Denmark	5.88	1.53	1.26	0.67	0.69	0.58
Finland	5.77	2.20	0.69	0.64	0.63	0.48
France	2.74	1.06	0.85	0.80	0.61	0.55
Germany	1.46	1.51	0.74	0.72	0.45	0.47
Italy	4.34	1.42	1.02	0.79	0.10	0.25
Japan	3.59	1.52	0.85	0.75	0.53	0.37
Netherlands	4.76	1.35	0.95	0.68	0.38	0.58
Sweden	3.92	1.62	0.99	0.52	0.57	0.36
Switzerland	3.41	1.67	0.62	0.79	0.10	0.04
United Kingdom	6.12	1.61	0.95	0.85	0.69	0.70
United States	3.79	1.50	0.80	0.90	0.57	0.51
Rich Countries	**4.00**	**1.48**	**0.88**	**0.68**	**0.46**	**0.39**

[1] The numbers are computed based on quarterly frequency data.
[2] The bold numbers represent each country group's average.
[3] Sources: BIS property price database, OECD national accounts.

less-related to housing. Following Shin (2021), I define housing consumption as expenditures on housing services and housing related utilities. In view of national accounts, housing consumption is the item of final consumption expenditure on housing, water, electricity, gas and other fuels that are included in the item of expenditures on services.[6] Non-housing consumption is defined as the remaining consumption, which is the sum of durable consumption and non-durable and service consumption other than housing consumption. Due to extremely limited availability of household expenditure data disaggregated by purpose, in particular, at a quarterly frequency, only four countries are available in my eighteen sample countries: two emerging countries (New Zealand and South Korea) and two rich countries (Canada and the United States). Note that the data-available countries show representativeness for each country group in terms of business cycle features (as noted in Table 2). The sample period is also adjusted considering data availability: 1975 Q1 to 2016 Q3 for South Korea, Canada, and the United States, and 1987 Q2 to 2016 Q3 for New Zealand.

Table 2 presents the key second moments of data-available countries with group average statistics in parentheses. House prices are more volatile in New Zealand and South Korea than in Canada and the United States and there is excess consumption volatility in New Zealand and South Korea, which are the stylized business cycle facts. It is notable that relative housing consumption volatility is significantly larger than non-housing consumption volatility in emerging countries ($\sigma_{HC}/\sigma_Y = 1.92 > 1.56 = \sigma_{NHC}/\sigma_Y$ for New Zealand; $\sigma_{HC}/\sigma_Y = 1.64 > 1.39 = \sigma_{NHC}/\sigma_Y$ for South Korea). In contrast, in both Canada and the United States, housing consumption volatility is smaller than non-housing consumption volatility ($\sigma_{HC}/\sigma_Y = 0.76 < 0.90 = \sigma_{NHC}/\sigma_Y$ for Canada; $\sigma_{HC}/\sigma_Y = 0.52 < 0.91 = \sigma_{NHC}/\sigma_Y$ for the United States). As a result, housing consumption volatilities for New Zealand and South Korea (1.92 and 1.64, respectively) are much greater than those for Canada and the United States (0.76 and 0.52, respectively). Starting from this finding, I extend the argument that house prices in emerging countries matter for excess consumption volatility. Specifically, higher housing consumption volatility in emerging countries needs to be considered one channel from house prices to consumption volatility. Finally, even if we consider higher housing consumption volatility, there is still excess volatility in the remaining part of consumption for emerging countries. That is, we can observe that the σ_{NHC}/σ_Y values for emerging countries are greater than unity, which requires reasoning beyond higher housing consumption volatility in explaining excess consumption volatility. As a result, I suggest "*Housing consumption is more volatile than non-housing consumption in emerging countries. Also, in emerging countries, excess volatility still holds for non-housing consumption*".

2.3. Intuition: Data to Model

Based on the evidence suggested above, here we consider what feature should be incorporated into the theoretical model; the evidence from correlations and disaggregated consumption implies that house price volatility matters for excess con-

[6] Shin (2021) disaggregates total consumption into two parts of housing consumption and non-housing consumption, and compares the volatilities of disaggregated consumption between emerging and rich countries based on both annual and quarterly frequency data with more sample countries. Shin (2021) suggests that housing consumption is more volatile than non-housing consumption in emerging countries with more countries and discusses them rigorously based on the differences in the fundamental related to housing. I clearly notice that the evidence from disaggregating total consumption in this manuscript is not the original contribution of this paper. Please refer to Shin (2021) for the detailed discussion.

Table 2
Business Cycles with Disaggregated Consumption.

	σ_{HP}	σ_Y	σ_C/σ_Y	σ_{HC}/σ_Y	σ_{NHC}/σ_Y
New Zealand	4.43	1.37	1.19	1.92	1.56
South Korea	5.16	2.41	1.19	1.64	1.39
(Emerging Countries)	(5.56)	(1.78)	(1.39)	(1.41)	(1.29)
Canada	3.52	1.42	0.74	0.76	0.90
United States	3.98	1.47	0.81	0.52	0.91
(Rich Countries)	(4.00)	(1.48)	(0.88)	(0.57)	(0.93)

[1] The numbers in the parentheses come from Table 1 for reference. They represent each country group's average. For the disaggregated consumption volatility, we referred to Shin (2021).
[2] Sources: BIS property price database, OECD national accounts, central banks, national statistics offices.

sumption volatility in emerging countries through both housing consumption and non-housing consumption. First, since housing consumption expenditure in national accounts is measured by the sum of the rental value of tenant-occupied housing and the imputed rental value of owner-occupied housing, the housing rental price emerges as a key factor linking house price volatility and housing consumption volatility. In fact, a positive correlation between house price volatility and rental price volatility is observed across countries in actual data, as Fig. 1 shows.[7] Additionally, the positive relationship is larger in magnitude in the case of emerging countries. Hence, I postulate that house prices affect housing consumption through a link between house prices and rental prices, which is relatively loose in emerging countries and relatively tight in rich countries, and I refer to it as a rental price pass-through.[8]

Next, we should consider the link between house prices and non-housing consumption in aggregate. Let us first take the well-known wealth effect of asset prices on consumption into account. The wealth effect means that a person becomes wealthier when the price of her own asset increases, which makes her consume more than before. However, houses are different from other assets for two reasons. First, people usually live in their house, directly valuing the services that their house provides. Therefore, the benefit of an increase in house prices is directly offset by an increase in the opportunity cost of housing services. Second, houses are not widely traded internationally. Thus, one country's homeowners in aggregate cannot realize their capital gains on houses to increase their consumption. In other words, all homeowners cannot simultaneously move out of homeownership, so the gain to a seller is also a loss to a buyer, who is also the country's consumer. This situation contrasts with capital gains on financial assets. As a result, there is no traditional wealth effect on consumption from house prices. However, there is another explanation in the literature; house prices may have a direct impact on non-housing consumption via the credit market effect, which is the so-called housing collateral effect. Specifically, this channel is rigorously investigated in Iacoviello (2005) and Berger et al. (2018) using U.S. data. Houses represent collateral for homeowners, and borrowing on a secured basis against housing collateral is generally less expensive than borrowing on an unsecured basis. Thus, an increase in house prices makes more collateral available to homeowners, which in turn may encourage them to borrow more (for instance, in the form of mortgage equity withdrawal in the United States) to finance consumption. The question that naturally follows is whether or not the collateral channel also works in emerging countries. For emerging countries in my sample, the average mortgage-to-GDP ratio, which is widely used as a mortgage depth indicator, is 53.1 percent.[9] This value is compatible with the average for rich countries, which is 58.6 percent. As a result, we can deduce that mortgages also matter in emerging countries. Considering the previous studies revealing the existence of housing collateral effect in advanced economies, the sufficient mortgage market in emerging countries implies that there also exists the effect that households' credit is affected by housing value changes in emerging economies.

3. The Models

In this section, I build a business cycle model for an emerging small open economy with housing and house prices that can capture the empirical evidence that I suggested in the previous section. This paper borrows the main building blocks from

[7] I extracted housing rent indices from CPI for each country. The rental price dataset is an unbalanced panel considering data availability. The sample countries with coverage and sources are as follows: Australia (1973 Q1-2016 Q3, Australian Bureau of Statistics), Canada (1970 Q1-2016 Q3, Statistics Canada), Denmark (2001 Q1-2016 Q3, Statistics Denmark), Finland (2000 Q1-2016 Q3, Statistics Finland), France (1990 Q1-2016 Q3, INSEE), Germany (1991 Q1-2016 Q3, DESTATIS), Italy (1996 Q1-2016 Q3, IStat), Japan (1970 Q1-2016 Q3, Statistics Bureau), New Zealand (1999 Q1-2016 Q3, Statistics New Zealand), South Africa (1970 Q1-2016 Q3, Statistics South Africa), South Korea (1975 Q1-2016 Q3, The Bank of Korea), Spain (2002 Q1-2016 Q3, INE), the United Kingdom (1996 Q1-2016 Q3, Institute for National Statistics), and the United States (1970 Q1-2016 Q3, BLS).

[8] This paper does not investigate why the relationship between rental prices and house prices is tighter in emerging countries than in rich countries. One plausible answer is based on cross-country variation in rental price stickiness. Different institutional environments and subjective factors across countries can lead to different magnitudes of rental price rigidity. As a result, rent is relatively flexible in emerging countries, while it is relatively sticky in rich countries, which makes the link between σ_{HP} and σ_ρ tighter for emerging countries and looser for rich countries. In that sense, this paper introduces the ad-hoc modelling of rental price rigidity, as described in Section 3.

[9] Badev et al. (2014) gauge the depth of each country's mortgage market by focusing on the total volume. They collect and show each country's outstanding mortgage debt relative to GDP on average for the period 2006 to 2010. The paper also presents housing loan penetration indices, which are defined by the percentage of the adult population with an outstanding loan to purchase a home. In terms of both indicators, the mortgage markets matter in emerging countries in my sample.

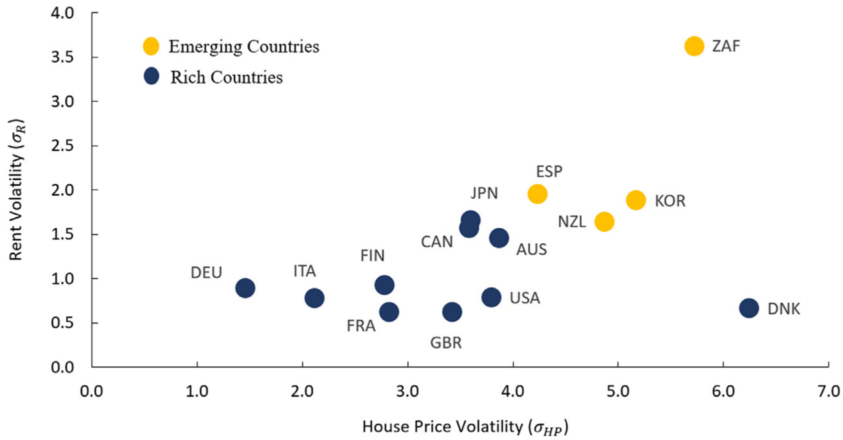

Fig. 1. House Price and Rent Volatility. Sources: Each country's central banks and statistics offices.

Iacoviello (2005). The model is essentially a business cycle model with financial friction that takes the form of a collateral constraint. Specifically, Iacoviello (2005) develops a monetary business cycle model with a borrowing constraint to explain the relationship between house prices and economic fluctuations in a large closed economy such as the United States. This paper has four primary variations. First, I discuss real economic fluctuations in this paper; thus, the model is a real business cycle model, not a nominal model, and it does not include monetary policy. Second, it is an open economy model; in particular, it considers a small open economy that categorizes most emerging countries. Third, this paper highlights housing consumption whose price measure is the housing rental price. I borrow an idea of rental price determination suggested by Sommer et al. (2013), but my model specification is more intuitive and straightforward in that the transaction of homogeneous housing rental service is simply made by the direct interaction between service providers and service demanders.[10] Finally, this paper focuses on the household's consumption responses to exogenous endowment and house price fluctuation, so the model is not a general equilibrium model but a partial equilibrium model without a production side, which is described below in detail.

Consider a discrete-time, infinite-horizon endowment economy, where households receive an endowment of goods at every period and there is no endogenous production.[11] The endowment is an exogenous and stochastic process, so it represents one source of uncertainty in the economy. The other source of uncertainty is the price of housing. It is also assumed to be an exogenously given stochastic process. Given the exogenous house price, an agent can purchase as many houses as she wants. That is, the economy has a demand-determined housing market, and housing supply is assumed to be perfectly elastic. Additionally, two sources of uncertainty are assumed not to affect each other.[12] Next, the model is a small open economy business cycle model that has an external debt-elastic interest rate following Schmitt-Grohé and Uribe (2003). The agent can access the international financial market, so she can borrow money at a given interest rate. In the case of the domestic bond market, the borrowing amount by debtors and the lending amount by creditors are exactly the same, which implies a zero domestic net credit supply in aggregate. As a result, all debt is international borrowing. Here, I assume that all unsecured lending behavior between lenders and borrowers is captured as domestic financial market transactions. Thus, the model does not consider an unsecured loan market. The only source of borrowing is foreign lenders, and lending against housing collateral should be secured.[13]

[10] Sommer et al. (2013) investigate the dynamics of house prices and rents in the U.S. through a theoretical model framework of housing tenure choice. In Sommer et al. (2013), rents are determined in a rental market where homeowners are rental service providers and tenants are demanders, and I just borrow this idea to model a housing rental market. While Sommer et al. (2013) introduce heterogeneous housing services by size and model endogenous homeownership, I assume the housing is homogeneous goods and homeownership rate is fixed.

[11] Labor supply and physical capital investment are intentionally ignored in the model as the endowment captures households' income from both sources. In the model, specifically, in the partial equilibrium model without production side, the endowment is introduced as an exogenous variable to figure out the relative influence of exogenous house price change through comparison, which is the main research question of this paper. Actually, if we assume a full-employment economy with no disutility from labor supply and a zero domestic net credit supply in aggregate, the implication of the model of this paper is exactly identical to that of the model with endogenous labor supply and capital investment.

[12] An independent process is a strong assumption because it is generally known that house prices are procyclical. In fact, we observe positive correlations between output and house prices in the data. However, the relationship between output and house prices is quite complicated beyond the observed correlation coefficients, so it should be discussed in a general equilibrium model with endogenous house prices. Nevertheless, I should note that I conducted numerical approximation with both independently estimated processes and jointly estimated processes, and found no significant different numerical results between them.

[13] Imagine commercial banks that finance funding from foreign creditors and domestic savers. Domestic savers are not necessarily homeowners; a domestic saver is either a homeowner or a renter. Banks extend unsecured lending to domestic borrowers, and its source is assumed to be domestic deposits without loss of generality. At the same time, banks extend secured lending against housing collateral, and its source is assumed to be foreign creditors. As a result, the model of this paper is simplified by removing financial intermediaries under the zero-sum domestic lending assumption.

This theoretical world is populated by two types of agents, *homeowners* and *renters*. The two agent groups are identical except in their ability to purchase houses. Homeowners decide housing investment in each period and access a secured loan against housing collateral from the international financial market, subject to a collateral constraint. In the case of renters, they behave as hand-to-mouth consumers because they do not access a financial market as they do not have collateralizable houses. Both agents share the same preference representation, which is non-separable between two goods (i.e., non-housing consumption goods and housing consumption goods), and the time unit of the model is a quarter.

Last, as clarified above, the model specifies financial friction homeowners faces as housing collateral constraint, taking the form of an inequality. We can interpret this borrowing constraint in two different dimensions; first, in view of time dimension, the constraint would be sometimes binding and sometimes unbinding; and second, in view of cross section, some proportion of homeowners faces a binding constraint and other proportion of homeowners does not face the binding constraint. Hence, this paper presents two models according to the above two interpretations. The first model is a two-agent model of homeowners with occasionally binding collateral constraint and renters. The second model is a three-agent model of unconstrained homeowners, constrained homeowners with binding borrowing constraint and renters.

3.1. The Two-Agent Model with Homeowners and Renters

In every period in the economy, there are ω homeowners and $(1-\omega)$ renters. The parameter ω controls both the share of homeowners and their share in consumption of goods. This parameter cannot change over time.

3.1.1. Homeowners

There are ω infinitely-lived identical homeowners with preferences described by the utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln X_t^O \tag{1}$$

where E_t denotes the expectation operator conditional on information available in period t, and X_t^O denotes a consumption aggregate. Note that the superscript "O" represents a homeowner. β is a subjective discount factor, and the periodic utility function takes the form of a logarithm. Homeowners consume two goods to increase their utility: non-housing consumption goods, c_t^O, and housing services, s_t^O. The two consumption goods compose a consumption bundle X_t^O, which is a constant elasticity of substitution aggregator:

$$X_t^O \equiv \left[\gamma \left(c_t^O\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_t^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\gamma > 0$ is the share of non-housing consumption goods in the consumption aggregator, and $\eta \geq 0$ is the intratemporal elasticity of substitution between non-housing consumption goods and housing services.

The homeowners maximize their lifetime utility subject to the sequence of budget constraints:

$$c_t^O + \rho_t s_t^O + q_t[h_t - (1-\delta)h_{t-1}] + d_t = y_t + \frac{d_{t+1}}{1+r_t} + \rho_t h_t \tag{3}$$

where q_t and ρ_t denote a house price and a housing rental price in real terms, respectively. h_t denotes housing stock that the owner is holding at period t, and δ is a quarterly depreciation rate. The homeowner spends funds on purchasing the housing stock at period t given an exogenous house price q_t, and at the same time, she receives as much funds as the current value of housing stock that she holds from the previous period after depreciation. As a result, the terms $h_t - (1-\delta)h_{t-1}$ represents a housing investment that the owner determines at period t. d_t denotes real debts whose maturity is time t, r_t denotes a real interest rate on loans between t and $t+1$, and y_t denotes the endowment the homeowner receives at period t. As seen on the right-hand side of the resource constraint, the funds for homeowners come from three sources: the endowment, international debt, and housing rental income. The last term on the right-hand side of Equation (3) is for "gross" housing rental income. Since linear technology allows the homeowner to produce exactly the same amount of housing services at no cost, the rental price times the amount of housing stock at period t is her rental income. Moving on to the left-hand side of the budget constraint, the uses of funds for homeowners are composed of four components: non-housing consumption given a numeraire price, housing consumption given a rental price, housing investment given the price of a house, and repayment of debts. In particular, the second term $\rho_t s_t^O$ is for the homeowner's expenditure on owner-occupied housing services. Since homeowners are providers of housing rental services (producing the amount of h_t at time t) and consumers of owner-occupied housing services (consuming the amount of s_t^O at time t), we can define "net" housing rental income as $\rho_t(h_t - s_t^O)$. Another aspect to mention regarding Equation (3) is the exogenous variables that the homeowners take as given. One source of uncertainty in the economy is the house price, q_t, and the other source is the endowment, y_t, as I mention above. I assume that the shocks follow the AR(1) process:

$$\ln q_{t+1} = \phi_q \ln q_t + \sigma_q \varepsilon_{q,t+1} \tag{4}$$

$$\ln y_{t+1} = \phi_y \ln y_t + \sigma_y \varepsilon_{y,t+1} \tag{5}$$

where ϕ_q and ϕ_y are the autoregressive parameters, σ_q and σ_y are the parameters for the standard deviation of innovation to the corresponding shock, and $\varepsilon_{q,t+1}$ and $\varepsilon_{y,t+1}$ are independently and identically distributed innovation with mean zero and variance one.

As in Iacoviello (2005) and Iacoviello and Neri (2010), I assume that homeowners face the borrowing limit. Formally,

$$d_{t+1} \leq m q_t h_t \tag{6}$$

where m represents a loan-to-value (LTV) ratio. Equation (6) implies that the borrowing amount is limited by the LTV ratio multiplied by the collateral value of housing stock that the owner is holding at period t. As a result, we can interpret the parameter m as the fraction of the house value used as collateral and the term $q_t h_t$ as the market value of collateral based on the current housing price (Kaplan et al. (2020), Sommer et al. (2013)).[14] Note that this constraint can be thought of as a down payment constraint for house purchase, but it also applies to any borrowing against home equity, not just to first-lien mortgages. Therefore, the use of d_{t+1} is not constrained. Next, I assume that the borrowing constraint is occasionally binding near the steady state. This is because I also assume the agents in the economy are sufficiently impatient. Formally, I assume $\beta(1+r^*) < 1$ where r^* is the steady-state level of the interest rate.

Given initial values $\{h_{-1}, d_0\}$, the homeowner chooses $\{c_t^O, s_t^O, h_t, d_{t+1}\}$ to maximize Equation (1) subject to Equations (3) and (6). The Lagrange multiplier for each constraint is defined as λ_t^O and $\lambda_t^O \mu_t$, respectively. The corresponding first order conditions are as follows:

$$\gamma \left(X_t^O\right)^{\frac{1-\eta}{\eta}} \left(c_t^O\right)^{-\frac{1}{\eta}} = \lambda_t^O \tag{7}$$

$$\frac{1-\gamma}{\gamma} \left(\frac{c_t^O}{s_t^O}\right)^{\frac{1}{\eta}} = \rho_t \tag{8}$$

$$\lambda_t^O q_t = \lambda_t^O \rho_t + \beta(1-\delta) E_t\left[\lambda_{t+1}^O q_{t+1}\right] + m \mu_t \lambda_t^O q_t \tag{9}$$

$$\lambda_t^O \left(\frac{1}{1+r_t} - \mu_t\right) = \beta E_t \lambda_{t+1}^O \tag{10}$$

Equations (7) and (8) are the first-order conditions with respect to the homeowner's non-housing consumption and housing services, respectively. In particular, Equation (8) represents the housing rental service supply condition because homeowners are housing rental service providers. Equation (9) is the first order condition with respect to the homeowner's housing stock, and it equates the marginal utility of non-housing consumption to the shadow value of housing stock. The latter relies on three components: a direct gain from additional rental income, an expected gain from a change in the value of a house realized in the next period, and a marginal utility of relaxing the collateral constraint. Last, Equation (10) is an Euler equation, but it is not a standard equation due to the existence of μ_t. When the collateral constraint is binding, the multiplier $\lambda_t^O \mu_t$ is positive. If the collateral constraint is slack, μ_t is zero, and we can obtain the standard Euler equation of marginal utilities between periods t and $t+1$.

3.1.2. Renters

Renters whose proportion in the economy is $(1-\omega)$ maximize their lifetime utility subject to a sequential budget constraint:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \ln X_t^R \tag{11}$$

$$\text{s.t. } c_t^R + \rho_t s_t^R = y_t \tag{12}$$

where the superscript "R" is for renters, so X_t^R denotes the renter's consumption aggregate, c_t^R denotes her non-housing consumption, and s_t^R denotes the housing services consumed. y_t is the endowment, and it follows the same process as the homeowner's endowment (5) because both agents face the same variation in endowment. The consumption aggregate X_t^R is defined as

[14] Several papers, including Kaplan et al. (2020) and Sommer et al. (2013), suggest that the collateral value in a borrowing constraint is based on the current house price, and I follow their view considering the collateral appraisal procedure using the current house price. At the same time, other papers have presented different views. For example, Iacoviello (2005), Iacoviello and Neri (2010), and Andrés et al. (2017) suggest that agents' maximum borrowing is given by the expected present value of their house times LTV ratio.

$$X_t^R \equiv \left[\gamma\left(c_t^R\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_t^R\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad (13)$$

Note that there is no intertemporal choice in the renter's problem. The only source of funds the renter has is the endowment that she receives at every period. As a result, the solution to the renter's optimization problem is equal to solving her static utility maximization problem period by period. Additionally, it is worth noting that parameters such as the discount factor, β, the share parameter, γ, and the elasticity parameter, η do not have superscripts, which implies that homeowners and renters are identical households with the exception of homeownership status.

The renter chooses optimal c_t^R and s_t^R at every period to maximize her periodic utility function $\ln X_t^R$ subject to her periodic resource constraint (12). By defining λ_t^R as the Lagrange multiplier for the resource constraint, the corresponding first order conditions as follows:

$$\gamma\left(X_t^R\right)^{\frac{1-\eta}{\eta}}\left(c_t^R\right)^{-\frac{1}{\eta}} = \lambda_t^R \quad (14)$$

$$\frac{1-\gamma}{\gamma}\left(\frac{c_t^R}{s_t^R}\right)^{\frac{1}{\eta}} = \rho_t \quad (15)$$

In Equation (14), the renter's marginal utility of non-housing consumption is equated to the shadow value of relaxing the budget constraint (12). Equation (15) represents the housing rental service demand condition in the economy because renters are housing rental service demanders in a housing rental market.

3.1.3. External Debt Elastic Interest Rate

Assume that the interest rate faced by homeowners, r_t, is increasing in the economy's average level of debt, which we denote by d_{t+1} as in Schmitt-Grohé and Uribe (2003). Formally, r_t is given by

$$r_t = r^* + \psi\left(e^{d_{t+1}-\bar{d}} - 1\right) \quad (16)$$

where the first term r^* is the steady-state level of the interest rate and the second term represents the interest rate premium. Note that $\psi > 0$ controls interest rate adjustment, and $\bar{d}$ denotes the steady-state level of aggregate debt. That is, the interest rate relies on a degree of distance of current debt from the steady state. This is an external debt elastic interest rate (EDEIR) because households take the cross-sectional average level of current debt as exogenously given.[15] Since $\omega d_{t+1} = d_{t+1}$ holds in equilibrium, the above specification (16) is rewritten by

$$r_t = r^* + \psi\left(e^{\omega d_{t+1}-\bar{d}} - 1\right) \quad (17)$$

3.1.4. Equilibrium

The clearing condition of the housing rental market is to equate the aggregate amount of rental supply to the aggregate amount of rental demand. Since the economy consists of ω homeowners and $(1-\omega)$ renters, the market clearing condition is:

$$\omega s_t^O + (1-\omega)s_t^R = \omega h_t \quad (18)$$

We derived the housing rental service supply condition (8) and demand condition (15) from the respective agent's optimization problem. As a result, we can solve for the rental market equilibrium using the supply and demand condition with the above market clearing condition (18). The equilibrium amount of housing rental services and the equilibrium housing rental prices are

$$s_t^R = \frac{\omega c_t^R}{c_t} h_t \quad (19)$$

$$\rho_t = \frac{1-\gamma}{\gamma}\left(\frac{c_t}{\omega h_t}\right)^{\frac{1}{\eta}} \quad (20)$$

where $c_t = \omega c_t^O + (1-\omega)c_t^R$, which is the total non-housing consumption.

Rigid Adjustment of Rental Prices I introduce rigid adjustment into rental prices in order to avoid extreme responses of rental prices and to prevent the model from generating extremely volatile housing consumption. In the data, housing rents generally fluctuate less than house prices, as Fig. 1 presents. Additionally, several papers, for example, Genesove (2003) and

[15] Uribe and Schmitt-Grohé (2017) also introduces the internal debt elastic interest rate (IDEIR) as an alternative way to induce stationarity. While the agents take the interest rate premium as a given in EDEIR specification, the model with an IDEIR assumes that the interest rate in increasing in the individual debt position, and so households internalize the effect that their borrowing choices have on the interest rate they face.

Verbrugge and Gallin (2019) suggest evidence for rental price rigidity. Based on data and the literature, the ad-hoc modeling idea of this paper is that rental prices want to stay at their steady-state level. I introduce an adjustment component that does not affect the steady state of the theoretical world. Specifically, I add the adjustment term to the equilibrium rental prices, or Equation (20), so the new equilibrium condition for rental prices is:

$$\rho_t = \frac{1-\gamma}{\gamma}\left(\frac{c_t}{\omega h_t}\right)^{\frac{1}{\eta}} \frac{1}{e^{\tau(\rho_t - \bar{\rho})}} \tag{21}$$

where τ is an adjustment parameter and $\bar{\rho}$ is a rigidity anchorage which is the steady-state level of rental prices. Adding the adjustment term on the equilibrium condition is interpreted as government intervention on private market equilibrium rental prices. After observing market clearing rental prices determined by the interaction between owners and renters, the government regulates the prices to prevent it from deviating far from its steady-state level. This type of government intervention is in line with Bond and Goldstein (2015) which introduces information friction on equilibrium prices, and Benigno et al. (2016) which designs a social planner problem subject to tax distortion on equilibrium prices.[16]

Aggregation We can construct the equilibrium process of aggregate variables such as total consumption, TC_t, housing consumption, HC_t, non-housing consumption, NHC_t, housing stock, H_t, international debt holdings, D_{t+1}, and gross income level, Y_t, by

$$HC_t = \rho_t \omega h_t \tag{22}$$

$$NHC_t = c_t \tag{23}$$

$$TC_t = HC_t + NHC_t \tag{24}$$

$$H_t = \omega h_t \tag{25}$$

$$D_{t+1} = \omega d_{t+1} \tag{26}$$

$$Y_t = y_t + \rho_t \omega h_t \tag{27}$$

Note that the last aggregate variable, Y_t, represents GDP measured by the income approach. Since, in this theoretical world, income is generated from endowment and housing rental service production, GDP is the summation of the two agents' endowments, $\omega y_t + (1-\omega)y_t$, and homeowners' rental income, $\rho_t \omega h_t$.

As a result, an equilibrium in the model is then a set of processes of 20 endogenous variables $\{c_t^O, c_t^R, c_t, h_t, s_t^O, s_t^R, d_{t+1}, X_t^O, X_t^R, \lambda_t^O, \lambda_t^R, \mu_t, \rho_t, r_t, TC_t, HC_t, NHC_t, H_t, D_{t+1}, Y_t\}_{t=0}^{\infty}$ satisfying (2) to (10), (12) to (15), (17) to (19), and (21) to (27), given the processes $\{q_t, y_t\}_{t=0}^{\infty}$ and the initial conditions $\{h_{-1}, d_0\}$. Please see the appendix for a full set of equilibrium conditions and the steady state of the economy.

3.2. The Three-Agent Model with Two Types of Homeowners and Renters

In order to compare the results to economies under less-tight credit constraints, we also study a version of the model with three households; in every period in the economy, there are ω_u unconstrained homeowners, ω_c constrained homeowners, and $(1-\omega_u-\omega_c)$ renters. The parameter ω_u and ω_c controls both the share of homeowners and their share in consumption of goods, and they do not change over time.

Note that the three-agent model has same environment as the two-agent model other than addition of the unconstrained owner's problem. While $\omega_u + \omega_c$ is equal to ω of the two-agent model, the three-agent model with two types of homeowners are never simple extension of the two-agent model; it is more complicated because a supply side of housing rental market, which is the important market homeowners and renters directly interact, is determined by two different agents, not a single agent. Note that, considering the cross-sectional interpretation of borrowing constraint, we assume that the ω_u homeowners do not face housing collateral constraint and the ω_c homeowners face binding borrowing constraint. Please refer to the appendix for the detailed set-up, equilibrium conditions, and the steady state of the economy.

[16] The simple idea of introducing adjustment of rental prices is based on the sequential responses of participants as follows; the uncontrolled rental prices are set in private housing rental market between homeowners and renters, and the uncontrolled prices are observed by the government; then the government preferring stable rental prices intervenes for the price not to deviate the prices from their steady state level by introducing adjustment term; finally, whole economy and each agent's responses are adjusted by the final rental prices controlled by the government. Note that the objective of introducing adjustment in the form of equilibrium condition intervention is to control excessive rental price volatility. In fact, the resulting consumption volatility is not significantly different regardless of existence of adjustment.

4. Results

In this section, I solve the baseline models numerically and show the results in various ways to learn how the models explain the linkage between consumption fluctuations and the given house price changes. Specifically, I solve the two-agent model through the piecewise linear approximation methodology suggested by Guerrieri and Iacoviello (2015) and the three-agent model through the perturbation method suggested by Uribe and Schmitt-Grohé (2017). In short, the exogenous housing price shock affects households' total consumption volatility primarily through housing collateral effect. Another explanation for the excessive volatility of total consumption is the housing rental price response induced by house price changes.

4.1. Calibration

To implement the numerical solution methods, I calibrate the parameters of the models introduced in the previous section. The target country is South Korea, and the time unit is one quarter as in the models.

I set the share of non-housing consumption parameter, $\gamma = 0.19$, to match the average housing consumption share in South Korea over the sample period, from 1975 Q1 to 2016 Q2, which is 15.2%. Together with the other calibrated parameters, it precisely determines the average housing consumption share. For the intratemporal elasticity of substitution between non-housing consumption goods and housing services, Flavin and Nakagawa (2008), Li et al. (2016), and Stokey (2009) estimated elasticity using household-level data, and they found that it is less than one. That is, non-housing consumption goods and housing services are complements according to the elasticity. Song (2011, 2012) and Davidoff and Yoshida (2013) also estimated an elasticity of less than one from macro-level aggregate consumption data.[17] In this paper, I pick $\eta = 0.35$, which is the estimate that Song (2012) provides, because the target country is Korea and Song's (2012) estimate of elasticity used aggregate consumption data of Korea. The steady-state level of interest rate, r^*, is set to 0.01 to match the average long-run interest rate per annum in Korea, 3.91%, over the sample period, which is from 1981 to 2016 considering data availability. The subjective discount factor is calibrated to $\beta = 0.95$ following Iacoviello (2005). Note that $\beta(1 + r^*) < 1$ because households are assumed to be sufficiently impatient. For the three-agent model, I set a higher discount factor for the unconstrained homeowners considering that the unconstrained homeowners are more patient than the constrained owners, i.e. $\beta_u > \beta_c$. Following Iacoviello and Neri (2010), $\beta_u = 0.99$ while $\beta_c = 0.95$. The quarterly housing depreciation rate, δ, is set to 0.0025 following Iacoviello (2005). In the case of the key parameter of the collateral constraint, m, the average loan-to-value (LTV) ratio for mortgages in Korea is considered. Due to the limited data availability for the actual LTV ratio in South Korea, I make use of the average over the last five years as a proxy. According to Bank of Korea, the average LTV ratio from 2012 to 2016 is 40 $\sim$ 60%. Additionally, Korean Financial Supervisory Services reported that the average LTV ratio was 53% as of the end of 2016. Hence, I set $m = 0.5$ for the baseline calibration. In fact, the regulatory LTV ratio can be one candidate for m, but there is a substantial gap between the regulatory ratio (70% since July 2014) and actual ratio in Korea. The homeowner proportion parameter ω is set to 0.55 to match the average homeownership rate in Korea, 55%. As the statistics for the homeownership rate are available since 1995 in Korea, it is also a proxy but it is fine because there is little variation in the homeownership rate over the sample period from 1995 to 2016.[18] For the three-agent model, we need to classify homeowners into the unconstrained owners or the constrained owners, which might be plausible only when we have all financial information of homeowners and their characteristics in South Korea. As an aggregate approach, this paper focuses on the proportion of homeowners with no mortgage, which is available statistics from the Survey of Household Finances and Living Conditions by Statistics Korea. The no-mortgage-homeowners include owners who paid off their mortgage. I postulate the homeowners with no mortgage are the unconstrained owners. According to Statistics Korea, the average proportion of no-mortgage-owners among homeowners is 38.4% (2012–2019). As a result, $\omega_u = 0.21$ and $\omega_c = 0.34$ for the three-agent model whose sum is equal to ω for the two-agent model. Next, I calibrate the premium adjustment parameter, $\psi = 0.000742$ following Schmitt-Grohé and Uribe (2003) and the rental price adjustment parameter for the two-agent and the three-agent model to match the ratio of rent volatility to house price volatility in South Korea which is 0.37.

Moving on to the parameters defining the stochastic processes of the exogenous driving forces, there are four parameters according to Equations (4) and (5). I conduct OLS estimations for the parameters using GDP and house price data of South Korea from 1975 Q1 to 2016 Q3, and the estimates are presented in Table 4. Both processes are quite persistent, with the

[17] There is little consensus in the literature on whether the elasticity is less than one or not. In contrast to the papers that I mentioned in the main text, Piazzesi et al. (2007) and Davis and Martin (2009) argue that $\eta > 1$ from the micro-level data estimation. Another branch of the literature ignores non-separable complementarity between non-housing consumption and housing services by using separable preference specification (Iacoviello (2005), Iacoviello and Neri (2010), Calza et al. (2013), Sun and Tsang (2017)), or simply assumes Cobb-Douglas preference, which implies that $\eta = 1$ (Sommer et al. (2013), Li and Yao (2007), Yang (2009), Aoki et al. (2004), Funke and Paetz (2013)).

[18] The homeownership rate in South Korea is a national statistic from two nationwide surveys: a biennial survey conducted by the Ministry of Land, Infrastructure and Transport (2006, 2008, 2010, 2012, 2014, 2016) and a census conducted Statistics Korea every five years (1995, 2000, 2005, 2010, 2015).

Table 3
Calibration.

Parameter			Value
Share of non-housing consumption		γ	0.19
Intratemporal elasticity of substitution		η	0.35
Steady state of interest rate		r^*	0.01
Subjective discount factor	**[Two-Agent]**	β	0.95
	[Three-Agent]	β_u	0.99
		β_c	0.95
Housing depreciation rate		δ	0.0025
Loan-to-value ratio		m	0.5
Homeowner proportion	**[Two-Agent]**	ω	0.55
	[Three-Agent]	ω_u	0.21
		ω_c	0.34
Interest rate premium adjustment		ψ	0.000742
Rental price adjustment	**[Two-Agent]**	τ	950
	[Three-Agent]		1,500

shock not vanishing until ten quarters after it occurs. According to the estimates, the volatility of each process is 2.41 for the endowment and 5.16 for house prices.[19]

4.2. Results

4.2.1. Selected Second Moments

In the previous section, I constructed several variables to observe the aggregate economy's business cycle features. Each aggregate variable is defined by the weighted summation of corresponding individual variables; in particular, GDP is calculated following the income approach by which GDP includes each group's endowment and homeowners' housing rental income. Then, I compute the second moments for the aggregate variables from 100,000 simulations to compare them with the business cycle facts observed in the actual data. The variables of primary interest are the relative consumption volatilities, and Table 5 shows the numbers for the data and models.

The total consumption volatility relative to GDP volatility for South Korea is 1.19, and the models generates 1.20 and 1.28, respectively. Although not presented in Table 5, total consumption volatility, σ_C, is larger for the models than for the data (3.13 for the two-agent model, 3.14 for the three-agent model and 2.87 for data), but the output volatility, σ_Y, for the models are 2.61 and 2.45, respectively, which are also larger than that for the data, 2.41.[20] As a result, the relative consumption volatility, σ_C/σ_Y, in the models successfully matches that observed in the data. Next, the disaggregated consumption volatilities generated in the models also capture the actual data qualitatively well. The key finding from the data is that housing consumption volatility is higher than non-housing consumption volatility in the case of emerging countries, and the models show large housing consumption volatility compared to non-housing consumption volatility ($\sigma_{HC}/\sigma_Y = 1.78$, $\sigma_{NHC}/\sigma_Y = 1.17$ for the two-agent model, and $\sigma_{HC}/\sigma_Y = 1.30$, $\sigma_{NHC}/\sigma_Y = 1.11$ for the three-agent model).[21]

Table 5 also presents the correlations of interest, which we discussed in Section 2, in South Korea and in the models. The aggregate consumption is positively correlated with output (0.59 for the two-agent model, 0.65 for the three-agent model) and house prices (0.42 for the two-agent model, 0.37 for the three-agent model) as in data ($\rho_{C,Y} = 0.70$, $\rho_{HP,C} = 0.38$). The correlations of house prices with disaggregate consumptions are positive mainly due to the two key channels the models feature, as we will see in detail later; the housing collateral effect makes non-housing consumption correlate with house prices (0.21 for the two-agent model, 0.27 for the three-agent model); the rental price pass-through makes housing consumption correlate with house prices (0.46 for the two-agent model, 0.21 for the three-agent model). While the simulated moments for $\rho_{HP,NHC}$ captures positive correlation observed in the data (0.38), $\rho_{HP,HC}$ in data shows acyclicality (0.03) which the models do not capture. Furthermore, although not presented in Table 5, the volatility of housing investment is reported

[19] We can consider an alternative specification for shock processes, which is VAR(1). We can assume that the law of motion of house prices and endowment is given by the following autoregressive process: $\begin{bmatrix} \ln q_{t+1} \\ \ln y_{t+1} \end{bmatrix} = A \begin{bmatrix} \ln q_t \\ \ln y_t \end{bmatrix} + \varepsilon_{t+1}$where A is 2 by 2 matrix and ε_{t+1} is a white noise of order 2 by 1 distributed $N(\phi, \Sigma_\varepsilon)$. The OLS estimation for the matrix A and Σ_ε results in the followed matrices:$A = \begin{bmatrix} 0.87 & 0.18 \\ 0.01 & 0.79 \end{bmatrix}, \Sigma_\varepsilon = \begin{bmatrix} 0.00021 & 0.00011 \\ 0.00011 & 0.00042 \end{bmatrix}$Note that the correlation coefficient parameters are quite small ($A_{12} = 0.18, A_{21} = 0.01$), and the innovation of each shock is also lowly correlated ($\Sigma_{\varepsilon,12} = \Sigma_{\varepsilon,21} = 0.00011$). As a matter of fact, the numerical results from the above VAR(1) specification are not quite different from the baseline case. Therefore, we can confirm that the results from the baseline specification of independent AR(1) shock processes are robust to the alternative one.

[20] One rational explanation for the excessive volatilities in consumption and GDP for the model compared to the data regards the assumption of exogenous house prices. Since the theoretical economy cannot adjust future house prices responding to current shocks, other variables such as consumption and GDP become more volatile.

[21] When we consider the two-agent model with always-binding borrowing constraint and solve it using the linear approximation, the model generates 1.22 for σ_C/σ_Y, 1.85 for σ_{HC}/σ_Y, and 1.21 for σ_{NHC}/σ_Y. The results for the selected second moments of interest are not significantly different from the baseline result although the values are slightly large. The main reason for similar results is that the interest rate movement responding to debt change is controlled by another friction of the external debt elastic interest rate in the model with always-binding borrowing constraint.

Table 4
Estimates for Shock Processes.

Parameter		OLS estimate
Persistence of house prices	ϕ_q	0.9093
Persistence of endowment	ϕ_y	0.7945
Standard deviation of innovation to house price shock	σ_q	0.0208
Standard deviation of innovation to endowment shock	σ_y	0.0144

Table 5
Selected Second Moments.

	Data[1]	Two-Agent Model[2]	Three-Agent Model[3]
σ_C/σ_Y	1.19	1.20	1.28
σ_{HC}/σ_Y	1.64	1.78	1.30
σ_{NHC}/σ_Y	1.39	1.17	1.11
σ_Y	2.41	2.61	2.45
σ_q	5.16	5.16	5.16
σ_ρ	1.89	1.88	1.88
$\rho_{C,Y}$	0.70	0.59	0.65
$\rho_{HP,Y}$	0.57	0.39	0.49
$\rho_{HP,C}$	0.38	0.42	0.37
$\rho_{HP,HC}$	0.03	0.46	0.21
$n\rho_{HP,NHC}$	0.38	0.21	0.27

[1] Seasonally adjusted HP filtered series ($\lambda = 1,600$), 1975 Q1-2016 Q3.
[2] Two-agent model with occasionally binding borrowing constraint, solved using a piecewise linear approximation method in Guerrieri and Iacoviello (2015).
[3] Three-agent model with unconstrained and constrained homeowners, solved using perturbation method of Uribe and Schmitt-Grohé (2017).

as follows: 12.01 in the data, 27.46 in the two-agent model and 22.47 in the three-agent model; the models generate housing investment that is more than twice as volatile as the data shows due to the setting of partial equilibrium model and the absence of housing investment adjustment cost.[22]

To see the influence of one shock compared to that of the other, I shut down each shock individually and then compute the second moments. The first (the forth) column of Table 6 is for the baseline case in the two-agent model (in the three-agent model) where both the housing price and endowment shock are effective. The second (the fifth) and third (last) columns are for the case where only one shock is effective in the two-agent model (in the three-agent model). I realized the exercise by assigning zero innovation shocks to the ineffective shock. For example, the column under the heading of shutdown of endowment shock shows the second moments when the innovation of the endowment process is zero. The exercise explicitly reveals the dominance of house price shocks in the theoretical models. The second moments for the case of shutting down the endowment shock in the two-agent model (in the three-agent model) are approximately twice (1.5 times) as large as those of shutting down the house price shock. In other words, the endowment shock is not solely able to generate a sufficiently volatile economy. On the other hand, the house price shock solely generates sufficient volatilities. Another point is related to the qualitative comparison. Regardless of the shutdown of one shock, all columns capture the finding of larger housing consumption volatility. Although the endowment shock alone is insufficient to generate a result similar to the baseline in terms of size, the third and last columns show that housing consumption volatility is still larger than non-housing consumption volatility. This result is because the intended channel within the model, which is a rental price pass-through, transmits the house price fluctuation, independent of endowment shock, into the rental price fluctuation. Since the rental price is the corresponding price for measuring housing consumption, housing consumption volatility still becomes larger than non-housing consumption volatility due to the rental price pass-through.

4.2.2. Impulse Responses to House Price Shock

To describe the mechanism behind the models, this subsection presents the impulse responses of aggregate variables in this economy—total consumption, housing consumption, non-housing consumption, housing stock, debt holdings, and housing rental price—to the house price shock in the two models.[23] The first panel of the first row of Fig. 2 indicates that one stan-

[22] In the models of this paper, the housing market is only driven by the housing demand of homeowners, and housing is a one-period asset in the market. It implies that there is no limitation on the housing supply and that homeowners can sell the old housing for the new housing purchase without any constraints. So homeowners purchase h_t after selling h_{t-1}, which is the housing investment in the theoretical world. As a result, the housing investment in the models of this paper is not conceptually consistent with the housing investment observed in the data. Also, in that sense, the models do not feature the housing investment adjustment cost term.

[23] In the case of the two-agent model, based on the piecewise linear approximation methodology of Guerrieri and Iacoviello (2015), the two regimes of binding constraints and unbinding constraints are shifting depending on shock innovation. The impulse responses when the constraints are binding (solved through linear approximation) and when the constraints are unbinding (solved through nonlinear approximation) are similar to those when the constraints are occasionally binding with slightly different magnitude.

Table 6
Shutdown of Each Shock.

	Two-Agent Model			Three-Agent Model		
	Baseline	Shutdown of		Baseline	Shutdown of	
		Endowment shock[1]	House Price Shock[2]		Endowment shock[1]	House Price Shock[2]
σ_C/σ_Y	1.20	1.54	0.68	1.28	1.38	0.93
σ_{HC}/σ_Y	1.78	2.30	0.79	1.30	1.44	0.96
σ_{NHC}/σ_Y	1.17	1.51	0.58	1.11	1.15	0.79

[1] The case in which only house price shock is effective.
[2] The case in which only endowment shock is effective.

dard deviation shock to house prices results in an approximately 1.1 percent point and 0.6 percent point increase in total consumption of the economy immediately after the shock, in the two-agent model and the three-agent model, respectively. Note that, in the case of the three-agent model, the response is hump-shaped, which has a peak of 1.0 percent point after a year. The increase in total consumption stems from the increase in both housing consumption and non-housing consumption, so I use two approaches to explain the consumption boom driven by housing prices. First, a positive house price shock relaxes homeowners' borrowing constraints through an increase in the value of a collateralizable asset, i.e., the housing stock they hold at the time of the shock. As a result, homeowners have more funding and achieve a higher level of non-housing consumption and housing investment. This is the reason why the non-housing consumption, borrowing amount, and housing stock increase together at the date of shock. Note that the hump-shapes in the three-agent model are owing to the existence of smoothing pressure by unconstrained homeowners. Second, the housing rental price responds also positively to the house price shock; it has a hump-shaped response in the two-agent model, and has decreasing-shape in the three-agent model. Recalling the equilibrium rental price, Equation (20), it relies on two endogenous variables: total non-housing consumption at time t, and housing stock at time t. Although both non-housing consumption and housing stock show a positive response to the house price shock, the magnitude of response and the speed of fall differ from each other, so the rental price presents a hump-shaped response in the two-agent model. Whereas, the response of non-housing consumption in the three-agent model is dominant when the shock occurs due to the existence of unconstrained owners, so the rental prices immediately increase and then continuously fall. After the shock occurs, the consumption expansion effect driven by an increase in house prices becomes weak due to the pressure of repayment of debt used for consumption increase. While the shock vanishes after ten quarters, the influence of the shock lingers for more than ten additional quarters.

In fact, the consumption boom after the increase in house prices is mainly led by homeowners. A positive house price shock stimulates homeowners to consume both housing services and non-housing consumption goods more than before. This effect is attributed to the improved credit condition by relaxing financial friction, as the (2,2) panel of Fig. 2 shows that the debt holding soars at the date of the shock. In contrast, a renter shows a minimal response to the positive house price shock. Rather, her utility decreases slightly due to the rise in the housing rental price. This originates from the property of renters as hand-to-mouth consumers; that is, in the face of a higher living cost, renters should adjust their consumption to a lower state in both housing services and non-housing consumption goods.[24] Please refer to the appendix for the agent-level impulse responses.

Next, in the case of endowment shock, a positive endowment shock leads to a consumption boom; both aggregate housing consumption and non-housing consumption increase, more housing investment and more borrowing take place, and the housing rental price increases. However, these effects are much smaller than the responses to the house price shock. The main reason is that the collateral constraint does not sufficiently relax with only an endowment shock unaccompanied by a change in the housing price. Another distinguishable difference from the responses to house price shocks is observed in agent-level responses; renter's responses are larger than homeowner's responses because the renter cannot access the international financial market and she allocates the exogenously given endowment to consumption between housing services and non-housing goods given the relative price of housing services. That is, the renter responds to the changes in endowment by a larger magnitude than the homeowners. Please refer to the appendix for the impulse responses to endowment shock.

4.3. House Price Volatility and Consumption Volatility

Under the baseline calibration, I compute the theoretical second moments: here, σ_C and σ_Y corresponding to different levels of house price standard deviation shock. Then, I plot the pairs of relative consumption volatility and house price volatility on the same plane, and a bold line (for the two-agent model) and a dashed line (for the three-agent model) in Fig. 3 are the results. According to the bold and dashed lines in Fig. 3, the more volatile the housing price is, the greater

[24] The renters in the theoretical world behave like hand-to-mouth consumers. As a result, the dynamics of the economy are dominated by the homeowners' behavior. However, it should be noted that the renters' consumption choice also affects the disaggregated consumption volatility through housing rental market.

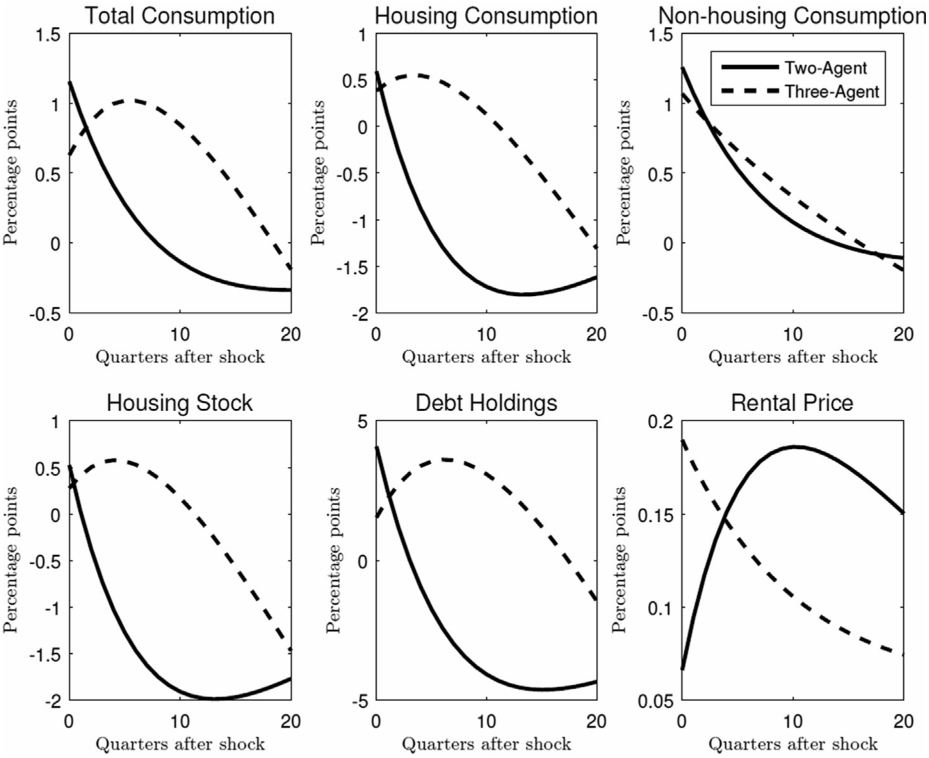

Fig. 2. Impulse Responses to House Price Shock. Notes: The impulse responses of total consumption, housing consumption, non-housing consumption, housing stock, debt holdings, and rental prices are expressed in percentage deviations from the deterministic steady state to 1-percent house price shock. The solid lines are from the two-agent model, and the dashed lines are from the three-agent model. The horizontal axis implies quarters after the shock occurs.

the volatility of relative consumption. The threshold of house price volatility for excess consumption volatility is approximately two in the two-agent model (and approximately three in the three-agent model). In other words, if house price volatility is greater than three, the excess consumption volatility is yielded by the model.[25] While the house price volatilities of most of sample countries, in Table 1, show the volatility above this threshold, I would like to underline that the models developed in this paper are for emerging economies. In other words, the models do not imply that excess consumption volatility has to occur in rich economies although house price volatility is greater than the threshold.

4.3.1. Key Mechanism 1: Collateral Constraint

Before moving on to the comparison analysis with a rich country, I describe key mechanisms of the baseline models. First, house prices matter for consumption volatility through the collateral effect, which is described in Subsections 2.3 and 4.2.2. To present the importance of the collateral constraint in the model, I compare the performance of the models with a collateral constraint and that of the model without a collateral constraint. Note that the two-agent model without borrowing constraint is mechanically equal to the three-agent model without constrained homeowners. As a dotted line in Fig. 3 presents, when I exclude a collateral constraint, the relative consumption volatility does not respond to the house price change. The σ_C/σ_Y stays at a low level regardless of a change in house price volatility in the model without a collateral constraint. Likewise, this paper asserts that financial friction in the form of housing collateral constraints is key in generating excess consumption volatility in emerging countries. As a result, this paper is in line with the literature that highlights financial friction in explaining the puzzle as in Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), and Alvarez-Parra et al. (2013). While previous works focus on frictions related to firm investment decision in the form of working capital or domestic interest rate

[25] When we plot the sample countries in the dataset, which includes 4 emerging and 14 rich countries, on a plane which takes σ_{HP} and σ_C/σ_Y as a horizontal and a vertical axis, respectively, we can observe a positive correlation between house price volatility and relative consumption volatility with a correlation coefficient is 0.74 ($R^2 = 0.55$). That is, a country whose house price volatility is high tends to have higher relative consumption volatility. The simulation results in Fig. 3 are consistent with a cross-country observation of positive correlation. Note that, however, the results of Fig. 3 are yielded by the theoretical model for emerging economy. That is, rigorously speaking, the simulation result should be interpreted as the positive relationship between two volatilities within emerging country group.

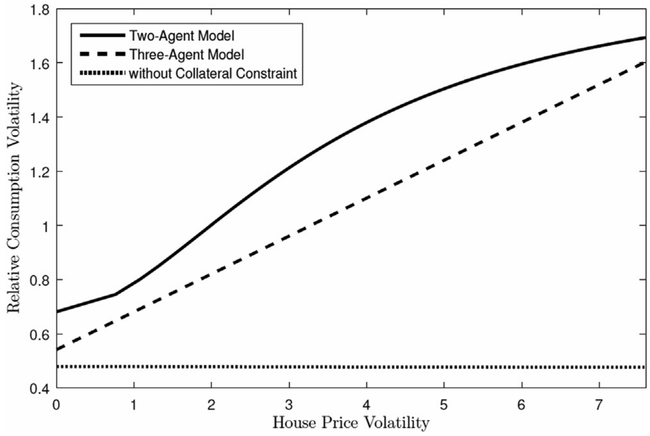

Fig. 3. House Price and Relative Consumption Volatility. Notes: The plots present the simulated total consumption volatility relative to output volatility with respect to different level of house price volatility. The solid lines are from the two-agent model, the dashed lines are from the three-agent model, and the dotted lines are from the model without the collateral constraint.

responses, this paper focuses on household consumption decisions related to housing by introducing collateral constraints. Please refer to the appendix for the detailed discussion regarding the validity of the mechanism of housing collateral constraints suggested in this paper.

Furthermore, since the renter does not access the international financial market and does not face a housing collateral constraint, the homeowner's responses are dominant in generating the positive relationship between house prices and consumption volatility, especially the excess consumption volatility observed in emerging countries. In particular, the homeownership rate, which is represented by the share parameter of the homeowner group, ω in the two-agent model (or $\omega_u + \omega_c$ in the three-agent model) matters for accounting for excess consumption volatility. Fig. 4 shows that the relative consumption volatility is an increasing function with respect to the homeownership rate.[26] Note that, in the three-agent model, since there does not exist local equilibrium when $\omega_u + \omega_c < 0.51$ and the equilibrium is locally indeterminate when $\omega_u + \omega_c > 0.76$ (with the fixed proportion of unconstrained homeowners, 38.4%), the simulated result is yielded on $\omega_u + \omega_c \in [0.51, 0.76]$. [27] In short, in emerging economies, a country whose homeownership share is higher tends to have higher relative consumption volatility.[28]

4.3.2. Key Mechanism 2: Rental Price Pass-Through

Second, the two-agent and three-agent model generate a strictly increasing relationship between housing price volatility and rental price volatility, as Fig. 5 indicates. This relationship implies that emerging countries whose house price volatility is large tend to have greater rent price volatility than rich countries. Even though the magnitude of correlation is larger in the models than in the data, Fig. 1 in Subsection 2.3 also explicitly presents a clear positive relationship across countries. The volatile housing rental price directly affects housing consumption volatility in the economy, which allows the model to replicate two findings observed in the data. First, housing consumption volatility is positively correlated with house price volatility, and second, non-housing consumption volatility also shows a positive correlation with house price volatility, but it is less than housing consumption volatility in the case of emerging countries (Fig. 6). This is what this paper names the rental price pass-through.

[26] The decreasing part of the bold line of Fig. 4 originates from the extremely low homeownership rate. The total consumption in the model is comprised of housing consumption and non-housing consumption, and the housing consumption is measured by the rental prices times the amount of housing services. Therefore, the extremely low homeownership rate, for example, ω close to zero implies the housing consumption is almost zero because there is no housing rental service transaction. As a result, the dominant role in generating total consumption volatility is non-housing consumption by renters. As ω becomes larger, the housing consumption recovers its role, and the non-housing consumption of homeowners becomes more important. But in the part of the sufficiently low homeownership rate, the falling speed of the power of non-housing consumption of renters is faster, so the simulation result shows a decreasing line in the part of $\omega = 0$ to 1. However, note that zero to 0.1 of homeownership rate is unrealistic.

[27] There does not exist local equilibrium when $\omega_u + \omega_c < 0.51$ because the three-agent model requires sufficient share of homeowners (or sufficiently small share of renters) to find the housing rental market equilibrium; if the share of homeowners is below the threshold, the aggregate rental supply decided by two types homeowners (according to sharing rules) cannot meet the aggregate high demand decided by large renter group. Also, the equilibrium is locally indeterminate when $\omega_u + \omega_c > 0.76$ because there exist multiple equilibria due to the large share of unconstrained homeowners. Note that homeownership rates in most countries in my sample fall in the region of determinancy of the three-agent model simulation, except two outlier countries (0.43 for Switzerland, and 0.78 for Spain). (Data sources: each country's statistics offices, central banks)

[28] Across countries including both emerging and rich countries, a positive relationship between relative consumption volatility and the homeownership rate is also observed with correlation coefficient of 0.28 ($R^2 = 0.08$), thus the model prediction is consistent with actual data. Note that, however, the result of Fig. 4 is generated by the theoretical model for emerging economy. That is, rigorously speaking, the comparison between model and data is effective only in emerging country group. (Data sources: each country's statistics offices, central banks)

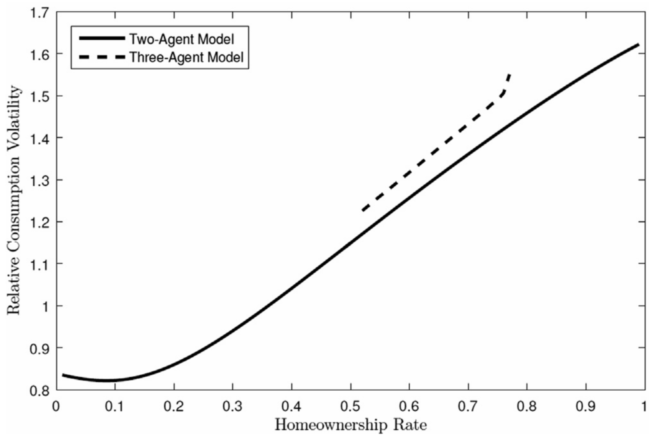

Fig. 4. Homeownership Rate and Consumption Volatility. Notes: The plots present the simulated total consumption volatility relative to output volatility with respect to different level of homeownership rate. The solid lines are from the two-agent model, and the dashed lines are from the three-agent model. In the case of the three-agent model, no local equilibrium exists when $\omega < 0.51$ and the equilibrium is locally indeterminate when $\omega > 0.76$.

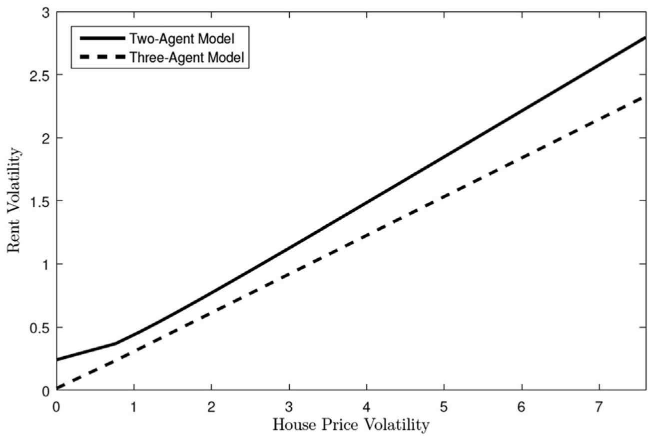

Fig. 5. House Price and Rent Volatility. Notes: The plots present the simulated rental price volatility with respect to different level of house price volatility. The solid lines are from the two-agent model, and the dashed lines are from the three-agent model.

Note that the degree of rental price pass-through is controlled by the rental price adjustment parameter, τ. If there is no adjustment, or $\tau = 0$, which means the rental prices are floating depending on the marginal rate of substitution between housing consumption and non-housing consumption, the models generate extremely high volatility in rental prices which is inconsistent with the data observation. The simulation result under $\tau = 0$ yields $\sigma_\rho = 8.67$ in the two-agent model ($\sigma_\rho = 10.55$ in the three-agent model) which is 1.7 times (2.0 times) more than house price volatility. More importantly, we can make use of the parameter τ as a source of a difference between emerging countries and rich countries, which is discussed in the next section.

5. Comparison: Emerging Country vs. Rich Country

In this section, I compare the baseline models with the counterfactual models, which are modified for an advanced small open economy, in particular, calibrated to Canadian economy. Considering the similarity of an emerging economy and a rich country as a small open economy, the counterfactual model environment features small variations compared to the baseline models. The main difference emerges in key parameters. First, η, which captures the intratemporal elasticity of substitution between housing consumption goods and non-housing consumption goods, is postulated to be larger in a rich country, which implies that the degree of complementarity between two goods are weaker in a rich country. Second, the rental price adjustment is stronger in rich countries as I discussed in Section 2.3 with Fig. 1, which implies a higher τ for the counterfactual case. Last, I hypothesize that homeowners in developed countries face looser credit conditions and thus a lower prob-

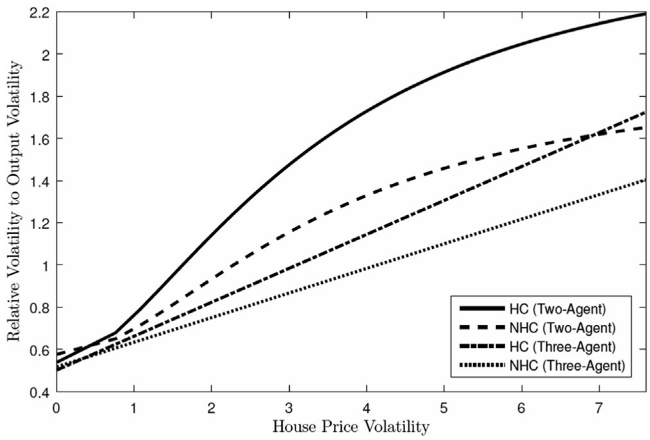

Fig. 6. House Price and Disaggregated Consumption Volatilities. Notes: The plots present the simulated disaggregate consumption volatility relative to output volatility with respect to different level of house price volatility. The solid (relative housing consumption volatility) and dashed lines (relative non-housing consumption volatility) are from the two-agent model, and the dotted (relative housing consumption volatility) and the dash-dot lines (relative non-housing consumption volatility) are from the three-agent model.

ability of hitting the borrowing constraint when we consider a smaller $\rho_{HP,C}$ observed in rich countries compared to emerging countries. However, there is a challenge in controlling the hitting frequency in the two-agent model with occasionally binding borrowing constraint.[29] For this reason, this section focuses on the comparison between emerging and rich economies using the three-agent model. In the three-agent model, we can reflect the lower hitting probability by adjusting the relative size of ω_u, a proportion of unconstrained homeowners. According to the baseline calibration strategy, this paper uses the proportion of homeowners with no mortgage as a proxy for ω_u. In the actual data, as described below, Canada has a higher share of no-mortgage-owners than South Korea.

5.1. Calibration for Rich Economy

Table 7 presents the parameter values for the counterfactual economy which is Canada. The share of non-housing consumption parameter γ for Canada is 0.53 to match the average housing consumption share over the sample period (23.0%), and the LTV ratio m is set to 0.9 reflecting the average loan-to-value ratio for mortgages in Canada. The homeownership rate is also larger for Canada, which is 0.69. For the counterfactual model corresponding to the three-agent model, $\omega_u = 0.31$ and $\omega_c = 0.38$ whose sum is equal to the homeownership rate. This calibration is based on the average proportion of homeowners with no mortgage, 44.5% (2012–2019, Survey of Financial Security, Statistics Canada). Note that, according to the calibration strategy, the share of unconstrained homeowners, $\omega_u/(\omega_u + \omega_c)$ is greater in Canada than in South Korea (0.45 and 0.38, respectively), which reflects the hypothesis of lower hitting probability of binding constraints in rich countries in view of cross section. More importantly, the key differences in the calibrated parameter compared to the baseline calibration for South Korea are in the intratemporal elasticity of substitution parameter η, and the rental price adjustment parameter τ. As I mentioned above, η for a rich country is assumed to be larger than that for an emerging country, so I set it to 0.74. Studies using the U.S. data recover the values for the elasticity parameter in the range of 0.15 and 0.74 (Flavin and Nakagawa (2008), Siegel (2008), Stokey (2009), Song (2011)). Considering the comparability of η between emerging and rich ecnomies, this paper counts on the estimation of Song (2011) because the author applied the identical elasticity estimation method in Song (2011) using the U.S. data and in Song (2012) using the Korea data. As a result, η for Canada is set to a relatively higher value ($\eta = 0.74$) and η for South Korea is set to relatively lower value ($\eta = 0.35$).[30] Next, τ is set to match the ratio of rent volatility to house price volatility in Canada, which is 0.39. Finally, the parameters for shock processes are estimated using Canadian data for house prices and GDP.

[29] The frequency of binding constraint is 76% when we solve the two-agent model using the piecewise linear approximation method suggested by Guerrieri and Iacoviello (2015). However, even if we allow the borrowing constraint for binding occasionally, any reasonable parameter changes for a rich economy cannot result in the lower hitting-probability in the present model environment; higher η combined with higher γ leads to higher hitting-frequency (79% under the calibration of Table 7); higher LTV ratio, m leads to higher hitting-frequency (91%); and higher β leads to higher hitting-frequency (95%). Therefore, in the case of the two-agent model, this paper compares the model with occasionally binding constraint for emerging economies and the model without borrowing constraint for rich economies, as an alternative. Please refer to the appendix for the details.

[30] As I previously discussed, there has been little consensus on the magnitude of the intratemporal elasticity of substitution. While several studies based on macro level aggregate consumption or asset price data frequently suggest a value greater than 1 (Piazzesi et al. (2007), Davis and Martin (2009), among them), this paper focuses on a value lower than 1 implying that households increase expenditure share on housing when house prices move up relative to prices of non-housing consumption.

Table 7
Calibration for Rich Economy (Canada).

Parameter		Value
Share of non-housing consumption	γ	0.53
Intratemporal elasticity of substitution	η	0.74
Steady state of interest rate	r^*	0.01
Subjective discount factor	β_u	0.99
	β_c	0.95
Housing depreciation rate	δ	0.0025
Loan-to-value ratio	m	0.9
Homeowner proportion	ω_u	0.31
	ω_c	0.38
Interest rate premium adjustment	ψ	0.000742
Rental price adjustment	τ	6,000
Persistence of house prices	ϕ_q	0.8463
Persistence of endowment	ϕ_y	0.8853
Standard deviation of innovation to house price shock	σ_q	0.0192
Standard deviation of innovation to endowment shock	σ_y	0.0066

5.2. Results for Rich Economy

The first two columns of Table 8 come from Table 5 in the previous section. The first column shows the quarterly business cycle moments in South Korea, and the second column under the heading of "Emerging" shows the moments yielded by the baseline three-agent model under the baseline calibration. As we discussed, the baseline model captures the actual data well. The third and the forth columns present the moments in Canadian data (under the heading "Canada") and in the model for the counterfactual economy (under the heading "Rich"). While the counterfactual model does not yield non-excessive consumption volatility ($\sigma_c/\sigma_Y = 1.02$), it shows good matching to the data in that housing consumption volatility is lower than non-housing consumption volatility and output is relatively stable. Comparing with the baseline model, the counterfactual one features lower volatility in total consumption and housing consumption, higher non-housing consumption volatility, and lower correlations of output and consumption with house prices. The differences are based on the composite effects from changes in values of key parameters; higher $\omega_u/(\omega_u + \omega_c)$ implies an increase in patient homeowners in the counterfactual economy; second, higher η and the additional sharing rule between two types of homeowners in the three-agent model contributes to the relative size of non-housing consumption volatility and housing consumption volatility; and third, higher τ weakens a rental price pass through.[31] As a result, lower house price volatility in Canada (with lower ϕ_q and σ_q), together with the different key parameter values, leads to less volatile consumption fluctuations than in South Korea. Nevertheless, the total consumption is not less volatile than output in "Rich". This is owing to m and $\omega_u + \omega_c$ greater in Canada than in South Korea, which implies there is a stronger housing collateral effect.[32] In fact, we can decrease the volatilities of consumption by decreasing the loan-to-value ratio for the constrained owners, m, and increasing the proportion of the unconstrained owners, $\omega_u/(\omega_u + \omega_c)$, as in the last column under the heading "Rich*" in Table 8.[33] Notice that the alternative calibrated values for m, ω_u, and ω_c are intentionally set in order to minimize the differences of relative consumption volatility and relative disaggregated consumption volatility between Canadian data and the counterfactual model.

Next, it is notable that the correlation coefficients between the counterfactual economy and the baseline economy support the evidence of correlations suggested in Section 2. That is, consumption is more correlated with house prices in the baseline case ($\rho_{HP,C} = 0.37$) than in the counterfactual case ($\rho_{HP,C} = 0.18$), while the correlation between output and house prices for the baseline case ($\rho_{HP,Y} = 0.49$) countries is at a similar level as that of the counterfactual case ($\rho_{HP,Y} = 0.44$). In the case of the correlations of disaggregated consumption with house prices, $\rho_{HP,NHC}$ is slightly greater than that for the baseline model (0.33 vs. 0.27), and $\rho_{HP,HC}$ is lower than that for the baseline model (0.04 vs. 0.21). The former implies that, in developed countries, non-housing consumption is more affected by house price fluctuation, which might be due to higher home-

[31] In order to check sensitivity of results to higher η in developed countries, I simulated the counterfactual model with the identical elasticity of substitution to South Korea ($\eta = 0.35$) implying higher magnitude of complementarity between non-housing consumption and housing services. Then there occurs excess consumption volatility ($\sigma_C/\sigma_Y = 1.45$) and higher non-housing consumption volatility ($\sigma_{NHC}/\sigma_Y = 1.51$), with a far greater housing consumption volatility ($\sigma_{HC}/\sigma_Y = 2.17$). However, it does not seem that a change in η alone controls the excessiveness of consumption volatility. This is because the simulation of the three-agent model with higher $\eta = 0.74$ (which is the identical elasticity of substitution to Canada) changes the volatilities in opposite direction; the relative volatilities of total consumption and non-housing consumption fall ($\sigma_C/\sigma_Y = 1.14, \sigma_{NHC}/\sigma_Y = 0.90$), and the relative volatility of housing consumption increases ($\sigma_{HC}/\sigma_Y = 1.87$).

[32] Actually, the two-agent model with occasionally binding borrowing constraint under the counterfactual calibration for Canada generates 4.28 for σ_C/σ_Y, 4.49 for σ_{HC}/σ_Y, and 4.91 for σ_{NHC}/σ_Y. So, the ratio of σ_{HC}/σ_{NHC} is less than one. However, there is a hugely excessive volatility in total consumption due to a higher LTV ratio, $m = 0.9$, and a higher homeownership rate, $\omega = 0.69$, in Canada.

[33] Note that the calibration strategy for m is to match an average LTV ratio and that for $\omega_u/(\omega_u + \omega_c)$ is to match a proportion of no-mortgage-homeowners. Therefore, to answer why "Rich" requires different values for m and $\omega_u/(\omega_u + \omega_c)$ for better performances, we can point out the validity of calibration strategies; the constrained households might face a lower LTV in reality due to their lower credit conditions, and the proportion of unconstrained homeowners might exceed or might be totally different from the simple statistic of proportion of homeowners with no mortgage in rich economies.

Table 8
Selected Second Moments: Emerging vs. Rich.

	Korea[1]	Emerging[2]	Canada[1]	Rich[2]	Rich*[3]
σ_C/σ_Y	1.19	1.28	0.74	1.02	0.75
σ_{HC}/σ_Y	1.64	1.30	0.76	0.99	0.76
σ_{NHC}/σ_Y	1.39	1.11	0.90	1.17	0.85
σ_Y	2.41	2.45	1.42	1.33	1.39
σ_q	5.16	5.16	3.52	3.52	3.52
σ_ρ	1.89	1.88	1.58	1.58	1.58
$\rho_{C,Y}$	0.70	0.65	0.49	0.56	0.56
$\rho_{HP,Y}$	0.57	0.51	0.35	0.23	0.44
$\rho_{HP,C}$	0.38	0.37	0.22	0.18	0.17
$\rho_{HP,HC}$	0.03	0.21	−0.22	0.04	0.04
$\rho_{HP,NHC}$	0.38	0.28	0.27	0.36	0.33

[1] Seasonally-adjusted HP-filtered series ($\lambda = 1,600$), 1975 Q1-2016 Q3.
[2] Three-agent model with unconstrained and constrained homeowners, solved using perturbation method of Uribe and Schmitt-Grohé (2017): "Emerging" uses the parameter values in Table 3, and "Rich" uses those in Table 7.
[3] Three-agent model under the calibration of Table 7 with alternative values for some parameters: $\omega_u = 0.45$, $\omega_c = 0.24$, and $m = 0.4$.

ownership rate and LTV ratio in the counterfactual economy. The latter is owing to tighter rental price adjustment in the counterfactual economy, but the theoretical model ($\rho_{HP,HC}$ in "Rich" column) fails to capture the negative correlation between housing consumption and house prices observed in the data ($\rho_{HP,HC}$ in "Canada" column).

In short, the three-agent model is encompassing emerging economies as well as rich economies, and we can clearly confirm that house price volatility is a source of the excess consumption volatility puzzle. Again notice that, however, if we take the model seriously, for excess consumption volatility in emerging economies to be due to a collateral channel we must also think that the channel is significantly weaker in developed countries, as we discuss through the exercise of "Rich*" in Table 8. Absent of another friction, through the lens of the model this may imply counterfactually low homeownership rates or a relatively small share of constrained households in developed economies. One could interpret this as evidence in favor of other channels not investigated in this paper for explaining excess consumption volatility.

6. Conclusion

With the aim of contributing to the literature explaining excess consumption volatility observed in emerging economies, this paper suggests a new angle regarding house prices. The importance of house prices in understanding the consumption fluctuation in one country should not be neglected because the share of housing service consumption is substantial in household expenditure, and a household's decision-making for housing purchase is strongly linked with consumption. Therefore, the stylized fact of higher house price volatility in emerging countries, which this paper suggests, is worth considering as a possible source of the explanation for an excess consumption volatility puzzle. As I discussed through the data findings and theoretical model frameworks, high house price volatility in emerging countries leads to excess volatility in total consumption through both the collateral credit channel and the rental price pass-through. The housing collateral effect explains why both non-housing consumption and housing service consumption fluctuated in response to changes in housing prices through changes in borrowing capacity, which is related to the value of a house as collateral. Rental price pass-through depends on a positive relationship between housing price and housing rental price, which explains the higher housing consumption volatility in emerging countries. Thus, housing price fluctuation affects higher total consumption volatility through higher housing consumption volatility.

An obvious limitation of this paper is that the theoretical models this paper develops does not perfectly fit the business cycles of rich economies. Even if the counterfactual model corresponding to the three-agent model with the unconstrained and constrained homeowners makes emerging countries and rich countries more comparable, than the counterfactual model corresponding to the two-agent model, it is not sufficient in that it requires justification of realistic values for key parameters. In fact, when we consider the housing collateral effect which is one of the essential mechanisms of the model and has also been largely discussed for developed economies in the literature, such as Iacoviello (2005) and Iacoviello and Neri (2010), the effect on household consumption volatility through the collateral credit channel might also be important in rich economies. While this paper confines its discussion to excess consumption volatility observed in emerging countries, an integrated general equilibrium model covering all countries can be developed by future studies to examine consumption volatilities and housing prices in an international context.

To conclude, we highlight that the higher house price volatility observed in emerging economies is also related to issues other than relative consumption volatility. In particular, the housing price is strongly related to wealth inequality or wealth redistribution in emerging countries. The link between housing prices and current accounts is another interesting issue to consider regarding emerging countries. My hope is that this paper will spark interest in housing price volatility in emerging countries.

CRediT authorship contribution statement

Wonmun Shin: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. House Price Dataset Construction

One contribution of Cesa-Bianchi et al. (2015) is the construction of a quarterly house price dataset for a substantial number of countries—33 emerging countries and 24 advanced countries. However, its sample coverage is from 1990 Q1 to 2012 Q4, thus providing house price time series with a maximum of 92 observations. Even in the case of emerging economies, approximately half of the countries cover less than 68 observations (starting from 1996 Q1), and the minimum coverage is from 2006 Q1 to 2012 Q4 with only 28 observations. This is one weakness of the Cesa-Bianchi et al. (2015) dataset in that it can cause a problem of overestimation of standard deviation. Next issue is the data heterogeneity from multiple sources of house price data. Cesa-Bianchi et al. (2015) mainly refers the Federal Reserve of Dallas international house price database, OECD house price database, and BIS property price dataset for the sources of their house price dataset. In addition, they collect data for several emerging countries from alternative sources other than OECD, BIS and Dallas Fed. As a result, the dataset in Cesa-Bianchi et al. (2015) is not liberated from inconsistency issue originated form a mixture of multiple source.

With the weakness of Cesa-Bianchi et al. (2015) data construction in mind, this paper constructs a cross-country house price dataset with efforts to obtain as long quarterly data as possible. Furthermore, I restrict the house price data source to only BIS property price database to control multiple source heterogeneity issue. In fact, the Dallas Fed international house price database is well-known and well- organized but is not a raw dataset whose house price indices are produced and updated based on its own methodology (Mack and Martinez-Garcia, 2011). OECD house price data are a raw dataset, but I conclude that the BIS dataset is superior to the OECD dataset in terms of the number of countries covered and the length of time series. Note that BIS property price dataset is also a collection of cross-country indices from each country, but they are raw data and so good starting point for dataset construction. Please refer to the Table A1 for each country's data source.

Table A1
Description and Sources of House Price Data.

Country	Indicator[1]	Source
Australia	Residential property prices (all dwellings, eight cities)	Australian Bureau of Statistics
Belgium	Residential property prices (all dwellings)	STABEL
Canada	Average price of existing homes, national residential average price	Canadian Real Estate Association
Denmark	Residential property prices (all dwellings, nationwide)	Statistics Denmark
Finland	Residential property prices (all dwellings)	Statistics Finland
France	Residential property prices (all dwellings)	INSEE
Germany	House price index, Residential property prices (all owner occupied dwellings)	Deutsche Bundesbank, Federal Statistical Office
Italy	Residential property prices (all dwellings)	Bank of Italy
Japan	Residential property prices (all dwellings)	Ministry of Land, Infrastructure, Transport and Tourism
Netherlands	Residential property prices (all dwellings)	Statistics Netherlands
New Zealand	Residential property prices (all dwellings)	Statistics New Zealand
South Africa	Residential property prices (all dwellings)	ABSA GROPU, First National Bank
South Korea	Residential property prices (all dwellings)	Bank of Korea
Spain	Residential property prices (all dwellings)	Bank of Spain
Sweden	Residential property prices (all dwellings)	Statistics Sweden
Switzerland	Unweighted average of owner occupied flats and houses nationwide	Wuest and Partner
United Kingdom	Residential property prices (all dwellings)	Office for National Statistics
United States	Residential property prices (existing dwellings)	Federal Reserves

[1] If there are more than two indicators are designated, the two indicators are used as supplements each other in constructing final data set.

Specifically, this paper undertakes considerable effort to ensure comparable house price indices across countries by confining the house price index; the index for residential buildings, covering all transactions, and the nationwide index or urban area index. In the case of several sources exist for one country, I chose one based on correlations, credibility of sources, and coverages. If there are missing values in time series, I interpolate under the assumption of a constant growth rate within the year.

Appendix B. Business Cycles with Alternative Detrending Methods

Table B1
Business Cycles in Emerging and Rich Countries (with Log-Quadratic Filter).

	σ_{HP}	σ_Y	σ_C/σ_Y	$\rho_{C,Y}$	$\rho_{HP,Y}$	$\rho_{HP,C}$
New Zealand	12.49	3.82	1.15	0.73	0.71	0.72
South Africa	18.02	5.17	0.69	0.61	0.75	0.69
South Korea	15.35	4.76	1.47	0.71	0.15	0.61
Spain	19.29	6.04	0.82	0.96	0.78	0.72
Emerging Countries	**16.29**	**4.95**	**1.03**	**0.75**	**0.60**	**0.68**
Australia	7.95	2.83	1.17	0.73	0.54	0.49
Belgium	11.32	2.48	0.88	0.35	0.69	0.29
Canada	9.30	3.29	0.64	0.75	0.13	0.18
Denmark	14.77	3.25	0.91	0.73	0.71	0.70
Finland	14.03	5.74	0.71	0.81	0.69	0.70
France	10.89	2.19	0.84	0.80	0.61	0.55
Germany	4.42	2.08	0.90	0.58	0.33	0.50
Italy	10.61	2.77	0.96	0.90	0.23	0.27
Japan	10.71	3.60	0.55	0.77	0.76	0.64
Netherlands	21.95	4.12	1.19	0.91	0.83	0.88
Sweden	11.60	3.78	0.83	0.82	0.54	0.48
Switzerland	12.24	2.94	0.58	0.59	0.21	0.16
United Kingdom	15.53	3.98	1.26	0.94	0.77	0.67
United States	11.59	3.01	1.04	0.96	0.58	0.56
Rich Countries	**11.92**	**3.29**	**0.89**	**0.76**	**0.54**	**0.50**

[1] The numbers are computed based on quarterly frequency data.
[2] The bold numbers represent each country group's average.
[3] Sources: BIS property price database, OECD national accounts.

Table B2
Business Cycles in Emerging and Rich Countries (with Log-Difference Filter).

	σ_{HP}	σ_Y	σ_C/σ_Y	$\rho_{C,Y}$	$\rho_{HP,Y}$	$\rho_{HP,C}$
New Zealand	4.22	1.58	2.40	0.22	0.16	0.82
South Africa	2.75	0.93	2.14	0.36	0.38	0.56
South Korea	2.51	1.65	1.21	0.69	0.34	0.37
Spain	2.82	0.79	1.24	0.58	0.38	0.40
Emerging Countries	**3.08**	**1.24**	**1.75**	**0.46**	**0.32**	**0.54**
Australia	2.07	0.92	1.08	0.42	0.32	0.27
Belgium	1.67	0.73	0.92	0.62	0.29	0.14
Canada	2.25	0.81	1.00	0.41	0.27	0.23
Denmark	2.71	1.08	1.61	0.45	0.35	0.26
Finland	2.53	1.46	1.07	0.60	0.38	0.32
France	1.35	0.56	1.24	0.65	0.38	0.30
Germany	0.85	0.98	1.06	0.57	0.29	0.35
Italy	2.00	0.84	1.21	0.53	0.29	0.31
Japan	1.52	1.06	1.02	0.70	0.42	0.34
Netherlands	2.43	1.13	0.97	0.35	0.30	0.40
Sweden	2.25	1.16	1.15	0.40	0.23	0.41
Switzerland	1.80	0.79	0.86	0.64	0.11	0.08
United Kingdom	2.70	0.90	1.19	0.62	0.49	0.51
United States	1.57	0.81	0.86	0.68	0.37	0.41
Rich Countries	**1.98**	**0.95**	**1.09**	**0.55**	**0.32**	**0.31**

[1] The numbers are computed based on quarterly frequency data.
[2] The bold numbers represent each country group's average.
[3] Sources: BIS property price database, OECD national accounts.

Appendix C. Housing Collateral Effect and Foreign Exposure

Note that, in the theoretical model, the homeowners raise funds for housing investment from international financial market, but not domestic financial market. Also, the prices of funding in the international market is determined by external debt elastic interest rates. This implies that the effect of housing collateral constraint is related to the domestic exposures on foreign counterpart. Therefore, in order to discuss the validity of the key mechanism of housing collateral effect in the model, I surveyed the data across sample countries, generating Table C1. The table presents the share of foreign currency debt for each sample country with varying coverage. On average, the share is higher in emerging economies compared to rich countries, which implies the economic fundamentals of emerging countries are consistent with the main mechanism this paper suggests.

To confirm the correlation between foreign exposure and house price volatility, we plot the sample countries on the plane whose horizontal axis is a share of foreign currency debt and vertical axis is house price volatility, as in Fig. C1. We observe a positive correlation between foreign exposure of financial institution and house price volatility across countries, with a correlation coefficient of around 0.5.

Table C1
Share of Foreign Currency Debt.

Country	Share (%)	Coverage	Country	Share (%)	Coverage
New Zealand	62.2	N.A.	France	24.1	1983–2018
South Africa	70.9	2009–2018	Germany	9.2	1983–2018
South Korea	82.2	2005–2018	Italy	12.9	1983–2018
Spain	19.0	1984–2018	Japan	45.2	1983–2018
Emerging	**58.6**		Netherlands	34.2	1983–2018
Australia	63.5	1997–2018	Sweden	63.2	1983–2018
Belgium	22.7	1983–2018	Switzerland	60.4	1983–2018
Canada	64.2	1983–2018	United Kingdom	68.6	1983–2018
Denmark	60.8	1983–2018	United States	8.8	1983–2018
Finland	29.0	1985–2018	**Rich**	**40.5**	

[1] The share is computed as cross-border bank claims in foreign currency over total cross-border bank claims. Cross-border bank lending is defined as foreign claims (all instruments) of all BIS reporting banks in all sectors, and deflated by US consumer price inflation. This calculation procedure follows that of Cesa-Bianchi et al. (2018). (In the case of New Zealand, BIS data is not available so this paper directly draws the number from Cesa-Bianchi et al. (2018). Their paper indicates their calculation is based on a confidential version of data.).
[2] The bold numbers represent each country group's average.

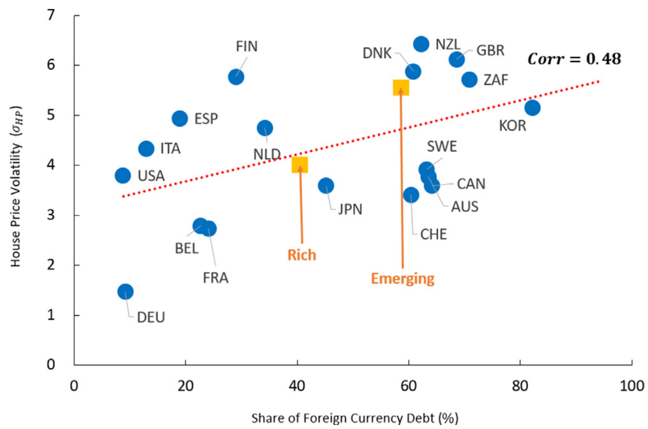

Fig. C1. Share of Foreign Currency Debt and House Price Volatility. Source: Author's calculation based on BIS data.

Appendix D. Impulse Responses

D.1 Agent-Level Impulse Responses to House Price Shock

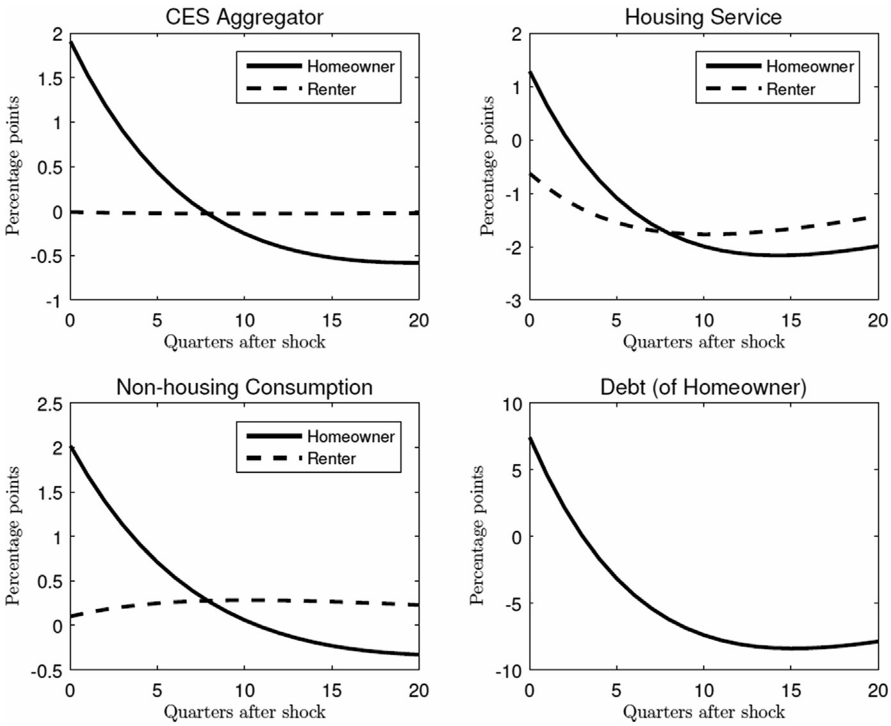

Fig. D1. Agent-Level IRF to House Price Shock (Two-Agent Model). Notes: The impulse responses of CES aggregator, housing services, non-housing consumption, and homeowner's debt are expressed in percentage deviations from the deterministic steady state to 1-percent house price shock. The solid lines are for homeowner's responses and the dashed lines are for renter's. The horizontal axis implies quarters after the shock occurs.

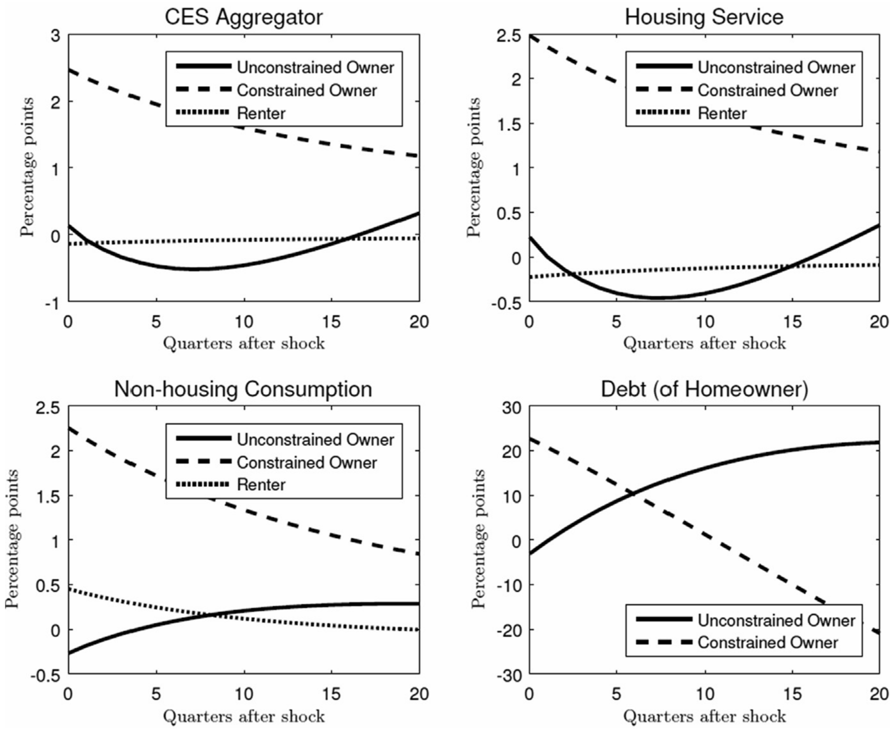

Fig. D2. Agent-Level IRF to House Price Shock (Three-Agent Model). Notes: The impulse responses of CES aggregator, housing services, non-housing consumption, and homeowner's debt are expressed in percentage deviations from the deterministic steady state to 1-percent house price shock. The solid lines are for unconstrained owner's responses, the dashed lines for constrained owner's, and the dotted lines for renter's. The horizontal axis implies quarters after the shock occurs.

D.2 Impulse Responses to Endowment Shock

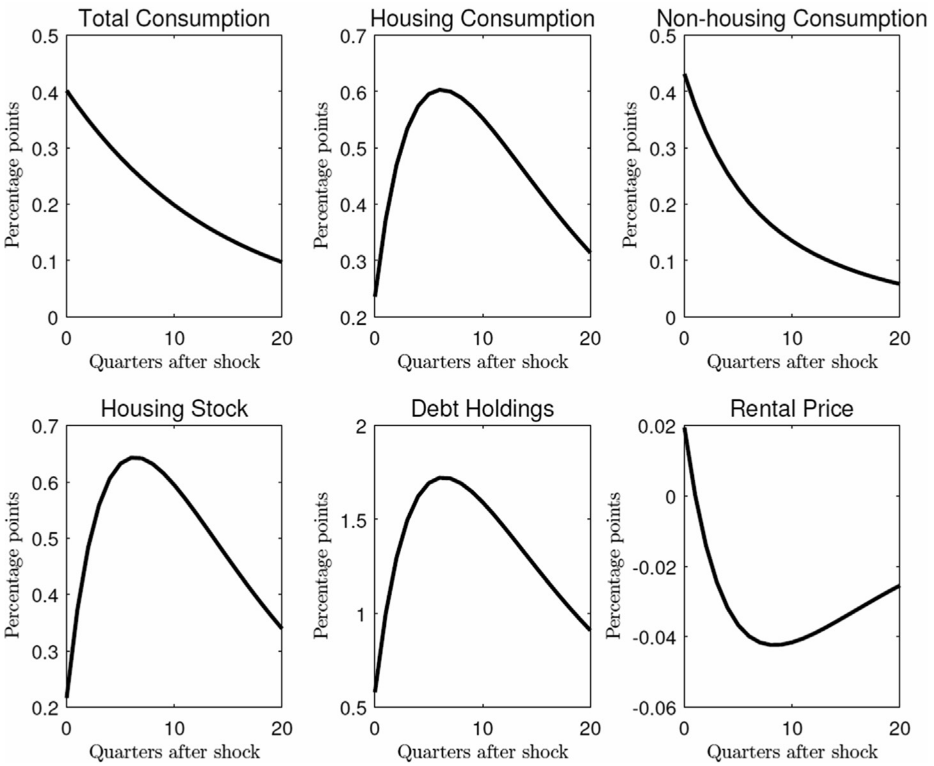

Fig. D3. Aggregate-Level IRF to Endowment Shock (Two-Agent Model). Notes: The impulse responses of total consumption, housing consumption, non-housing consumption, housing stock, debt holdings, and rental prices are expressed in percentage deviations from the deterministic steady state to 1-percent endowment shock. The horizontal axis implies quarters after the shock occurs.

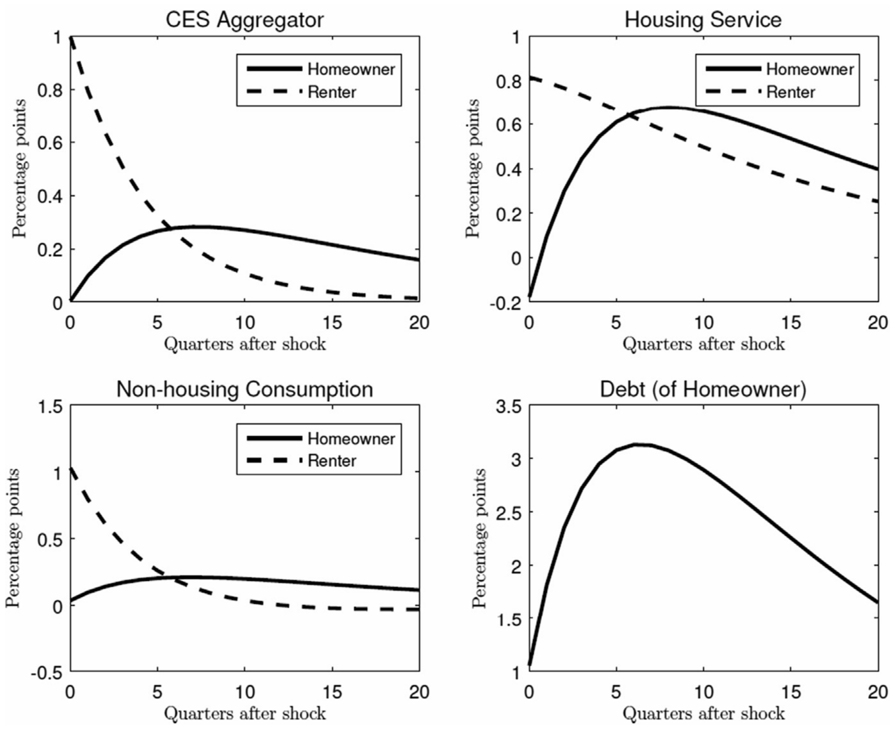

Fig. D4. Agent-Level IRF to Endowment Shock (Two-Agent Model). Notes: The impulse responses of CES aggregator, housing services, non-housing consumption, and homeowner's debt are expressed in percentage deviations from the deterministic steady state to 1-percent endowment shock. The solid lines are for homeowner's responses and the dashed lines are for renter's. The horizontal axis implies quarters after the shock occurs.

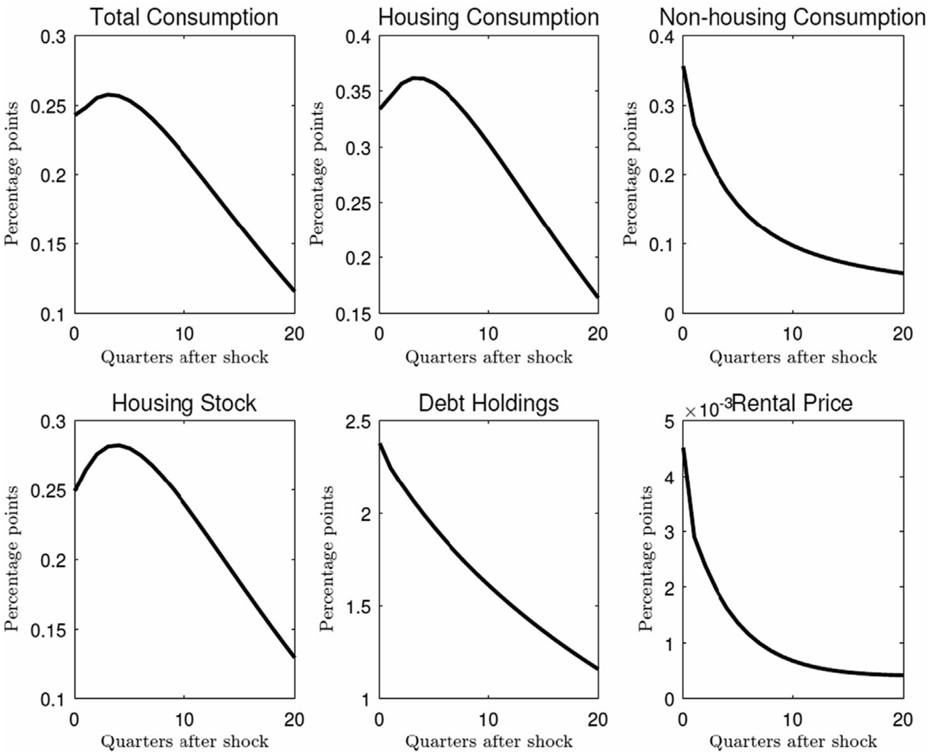

Fig. D5. Aggregate-Level IRF to Endowment Shock (Three-Agent Model). Notes: The impulse responses of total consumption, housing consumption, non-housing consumption, housing stock, debt holdings, and rental prices are expressed in percentage deviations from the deterministic steady state to 1-percent endowment shock. The horizontal axis implies quarters after the shock occurs.

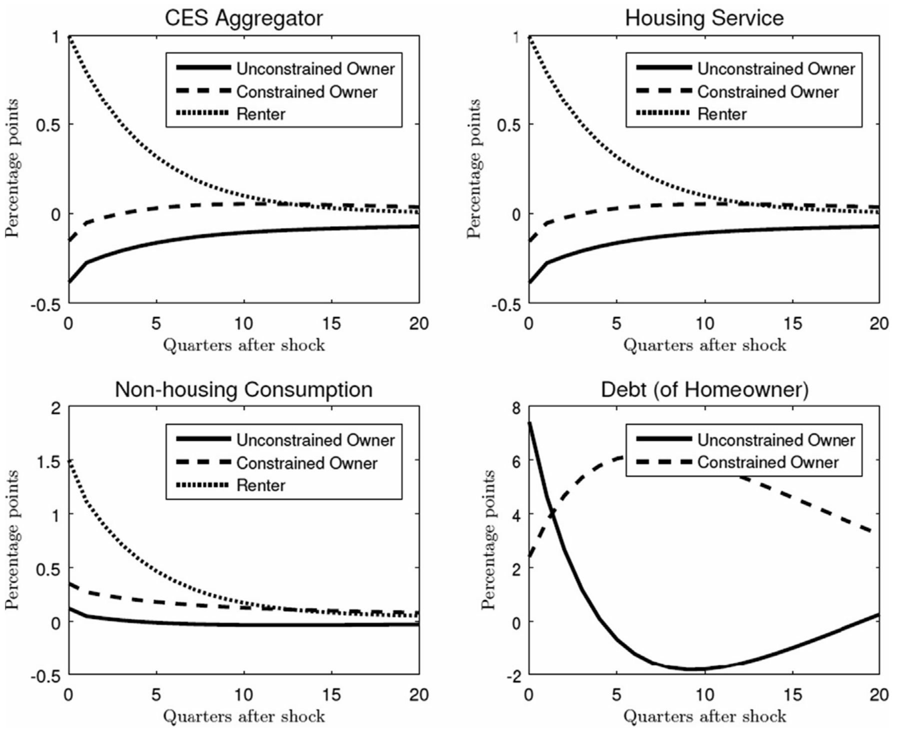

Fig. D6. Agent-Level IRF to Endowment Shock (Three-Agent Model). Notes: The impulse responses of CES aggregator, housing services, non-housing consumption, and homeowner's debt are expressed in percentage deviations from the deterministic steady state to 1-percent endowment shock. The solid lines are for unconstrained owner's responses, the dashed lines for constrained owner's, and the dotted lines for renter's. The horizontal axis implies quarters after the shock occurs.

Appendix E. Two-Agent Model: Equilibrium Conditions and the Steady States

E.1 Full Equilibrium Conditions

An equilibrium in the model is a set of processes of $\{c_t^O, c_t^R, c_t, h_t, s_t^O, s_t^R, d_{t+1}, X_t^O, X_t^R, \lambda_t^O, \lambda_t^R, \mu_t, \rho_t, r_t, TC_t, HC_t, NHC_t, H_t, D_{t+1}, Y_t\}_{t=0}^{\infty}$ satisfying (2) to (10), (12) to (15), (17) to (19), and (21) to (27), given the processes $\{q_t, y_t\}_{t=0}^{\infty}$ and the initial condition $\{h_{-1}, d_0\}$. Therefore, there are 20 endogenous variables, and we need 20 equations to determine equilibrium. Also, we need 2 equations which specify exogenous variable processes.

$$\gamma\left(X_t^O\right)^{\frac{1-\eta}{\eta}}\left(c_t^O\right)^{-\frac{1}{\eta}} = \lambda_t^O \tag{E.1}$$

$$\gamma\left(X_t^R\right)^{\frac{1-\eta}{\eta}}\left(c_t^R\right)^{-\frac{1}{\eta}} = \lambda_t^R \tag{E.2}$$

$$c_t = \omega c_t^O + (1-\omega)c_t^R \tag{E.3}$$

$$\rho_t = \frac{1-\gamma}{\gamma}\left(\frac{c_t}{\omega h_t}\right)^{\frac{1}{\eta}}\frac{1}{e^{\tau(\rho_t-\bar{\rho})}} \tag{E.4}$$

$$s_t^R = \frac{\omega c_t^R}{c_t}h_t \tag{E.5}$$

$$\omega s_t^O + (1-\omega)s_t^R = \omega h_t \tag{E.6}$$

$$\lambda_t^O\left(\frac{1}{1+r_t} - \mu_t\right) = \beta E_t \lambda_{t+1}^O \tag{E.7}$$

$$\lambda_t^O q_t = \lambda_t^O \rho_t + \beta(1-\delta)E_t\left[\lambda_{t+1}^O q_{t+1}\right] + m\mu_t\lambda_t^O q_t \tag{E.8}$$

$$X_t^O \equiv \left[\gamma\left(c_t^O\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_t^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{E.9}$$

$$X_t^R \equiv \left[\gamma\left(c_t^R\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_t^R\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{E.10}$$

$$c_t^O + \rho_t s_t^O + q_t[h_t - (1-\delta)h_{t-1}] + d_t = y_t + \frac{d_{t+1}}{1+r_t} + \rho_t h_t \tag{E.11}$$

$$d_{t+1} \le m q_t h_t \tag{E.12}$$

$$c_t^R + \rho_t s_t^R = y_t \tag{E.13}$$

$$r_t = r^* + \psi\left(e^{\omega d_{t+1}-\bar{d}} - 1\right) \tag{E.14}$$

$$HC_t = \rho_t \omega h_t \tag{E.15}$$

$$NHC_t = c_t \tag{E.16}$$

$$TC_t = HC_t + NHC_t = c_t + \rho_t\omega h_t \tag{E.17}$$

$$H_t = \omega h_t \tag{E.18}$$

$$D_{t+1} = \omega d_{t+1} \tag{E.19}$$

$$Y_t = y_t + \rho_t\omega h_t \tag{E.20}$$

$$\ln q_{t+1} = \phi_q \ln q_t + \sigma_q \varepsilon_{q,t+1} \tag{E.21}$$

$$\ln y_{t+1} = \phi_y \ln y_t + \sigma_y \varepsilon_{y,t+1} \tag{E.22}$$

Equation (E.1) and (E.2) are FOCs with respect to each agent's non-housing consumption, equation (E.3) is a definition equation for total non-housing consumption, equation (E.4) is the equilibrium rental price, equation (E.5) is the equilibrium rental service amount, equation (E.6) is the rental market clearing condition, equation (E.7) is the Euler equation earned by differentiating with respect to d_{t+1}, equation (E.8) is FOC with respect to housing purchase decision by homeowner, equation (E.9) and (E.10) are CES aggregators of non-housing consumption and housing service equation for each agent group. Equation (E.11) and (E.12) are a homeowner's resource constraint and a borrowing constraint, respectively. Equation (E.13) is a renter's budget constraint. Equation (E.14) is EDEIR. Equation (E.15) to (E.20) represent the definition of each aggregate variable. Equation (E.21) and (E.22) are exogenous shock processes for house price and endowment, respectively.

E.2 The Steady States of the Economy

First of all, r^* is the steady state level of interest rate, by definition. From (E.21) and (E.22),

$$q^* = 1$$

$$y^* = 1$$

From (E.7),

$$\mu^* = \frac{1}{1+r^*} - \beta$$

Since $\beta(1+r^*) < 1$ is assumed, the shadow price parameter μ^* at the steady state is greater than zero. As we know μ^*, we can earn the steady state level of rental price, ρ^*, from (E.8):

$$\rho^* = 1 - \beta(1-\delta) - m\mu^*$$

Next, from (E.12),

$$d^* = mh^* \tag{E.i}$$

Note that the parameter for the steady state level of aggregate debt, $\bar{d}$ must be equal to ωd^* by (E.14). Moving on to (E.4), we can earn the expression for the term ωh^*:

$$\omega h^* = \left(\frac{\gamma}{1-\gamma}\rho^*\right)^{-\eta} c^*$$

Using the above expression, s_R^* can be re-written from (E.5) by

$$s_R^* = \left(\frac{\gamma}{1-\gamma}\right)^{-\eta} \rho^{*-\eta} c_R^* \tag{E.ii}$$

By plugging (E.ii) into (E.13), we get the steady state level of renter's non-housing consumption, c_R^*:

$$c_R^* = \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}} \tag{E.iii}$$

We earn the expression for c_O^* by rearranging (E.3) using (E.4):

$$c_O^* = \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*\eta} h^* - \frac{1-\omega}{\omega} c_R^* \tag{E.iv}$$

Also, we earn the expression for s_O^* by rearranging (E.6) using (E.ii):

$$s_O^* = h^* - \frac{1-\omega}{\omega}\left(\frac{\gamma}{1-\gamma}\right)^{-\eta} \rho^{*-\eta} c_R^* \tag{E.v}$$

By putting (E.i), (E.iii), (E.iv), and (E.v) into (E.11),

$$h^* = \frac{1}{\omega}\left[\left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*\eta} + \delta + \frac{mr^*}{1+r^*}\right]^{-1}$$

Therefore, we can get a set of constant sequences $c_t^O = c_O^* > 0$, $c_t^R = c_R^* > 0$, $c_t = c^* > 0$, $h_{t-1} = h_t = h^* > 0$, $s_t^O = s_O^* > 0$, $s_t^R = s_R^* > 0$, $d_t = d_{t+1} = d^* > 0$, $X_t^O = X_O^* > 0$, $X_t^R = X_R^* > 0$, $\lambda_t^O = \lambda_O^* > 0$, $\lambda_t^R = \lambda_R^* > 0$, $\mu_t = \mu^* > 0$, $\rho_t = \rho^*$, $r_t = r^*$, $TC_t = TC^* > 0$, $HC_t = HC^* > 0$, $NHC_t = NHC^* > 0$, $H_t = H^* > 0$, $D_{t+1} = D^* > 0$, and $Y_t = Y^* > 0$. The multiplier for the collateral constraint should be positive since the constraint is binding at the steady state. House price and endowment are also constant at the steady state, $q_t = q^*$, and $y_t = y^*$.

$$q^* = 1$$

$$y^* = 1$$

$$d^* = mh^*$$

$$\mu^* = \frac{1}{1+r^*} - \beta > 0$$

$$\rho^* = 1 - \beta(1-\delta) - m\mu^* > 0$$

$$h^* = \frac{1}{\omega}\left[\left(\frac{1-\gamma}{\gamma}\right)^{\eta}\rho^{*\eta} + \delta + \frac{mr^*}{1+r^*}\right]^{-1}$$

$$c_R^* = \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta}\rho^{*1-\eta}} > 0$$

$$c_O^* = \left(\frac{1-\gamma}{\gamma}\right)^{\eta}\rho^{*\eta}h^* - \frac{1-\omega}{\omega}c_R^* > 0$$

$$c^* = \omega c_O^* + (1-\omega)c_R^*$$

$$s_O^* = h^* - \frac{1-\omega}{\omega}\left(\frac{1-\gamma}{\gamma}\right)^{\eta}\frac{c_R^*}{\rho^{*\eta}}$$

$$s_R^* = \left(\frac{1-\gamma}{\gamma}\right)^{\eta}\frac{c_R^*}{\rho^{*\eta}}$$

$$X_O^* \equiv \left[\gamma\left(c_O^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_O^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$X_R^* \equiv \left[\gamma\left(c_R^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_R^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\lambda_O^* = \gamma\left(X_O^*\right)^{\frac{1-\eta}{\eta}}\left(c_O^*\right)^{-\frac{1}{\eta}}$$

$$\lambda_R^* = \gamma\left(X_R^*\right)^{\frac{1-\eta}{\eta}}\left(c_R^*\right)^{-\frac{1}{\eta}}$$

$$HC^* = \rho^*\omega h^*$$

$$NHC^* = c^*$$

$$TC^* = c^* + \rho^*\omega h^*$$

$$H^* = \omega h^*$$

$$D^* = \omega d^*$$

$$Y^* = 1 + \rho^*\omega h^*$$

Appendix F. Three-Agent Model: Model Set-Up, Equilibrium Conditions and the Steady States

F.1 The Three-Agent Model with Two Types of Homeowners

F.1.1 Unconstrained Homeowners

There are ω_u infinitely-lived identical homeowners with preferences described by the utility function:

$$\max E_0 \sum_{t=0}^{\infty} \beta_u^t \ln X_{u,t}^O \text{ where } X_{u,t}^O \equiv \left[\gamma\left(c_{u,t}^O\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_{u,t}^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.1}$$

$$\text{s.t. } c_{u,t}^O + \rho_t s_{u,t}^O + q_t[h_{u,t} - (1-\delta)h_{u,t-1}] + d_{u,t} = y_t + \frac{d_{u,t+1}}{1+r_t} + \rho_t h_{u,t} \tag{F.2}$$

where the superscript "O" represents a homeowner as in the two-agent model, and the subscript "u' an unconstrained owner. The description for all variables in the unconstrained homeowner's problem is same as in the two-agent model: β_u stands for a subjective discount factor, $X_{u,t}^O$ for consumption bundle, $c_{u,t}^O$ for non-housing consumption, $s_{u,t}^O$ for housing services, $h_{u,t}$ for housing stock, $d_{u,t+1}$ for borrowing. Note that the unconstrained owners do not faces borrowing constraint. Also, the sources of uncertainty in the economy follow the AR(1) processes as in the two-agent model:

$$\ln q_{t+1} = \phi_q \ln q_t + \sigma_q \varepsilon_{q,t+1} \tag{F.3}$$

$$\ln y_{t+1} = \phi_y \ln y_t + \sigma_y \varepsilon_{y,t+1} \tag{F.4}$$

Given initial values $\{h_{u,-1}, d_{u,0}\}$, the unconstrained homeowner chooses $\{c_{u,t}^O, s_{u,t}^O, h_{u,t}, d_{u,t+1}\}$ to maximize Equation (F.1) subject to Equations (F.2). The Lagrange multiplier for the resource constraint is defined as $\lambda_{u,t}^O$. The corresponding first order conditions are as follows:

$$\gamma\left(X_{u,t}^O\right)^{\frac{1-\eta}{\eta}}\left(c_{u,t}^O\right)^{-\frac{1}{\eta}} = \lambda_{u,t}^O \tag{F.5}$$

$$\frac{1-\gamma}{\gamma}\left(\frac{c_{u,t}^O}{s_{u,t}^O}\right)^{\frac{1}{\eta}} = \rho_t \tag{F.6}$$

$$\lambda_{u,t}^O q_t = \lambda_{u,t}^O \rho_t + \beta_u(1-\delta)E_t\left[\lambda_{u,t+1}^O q_{t+1}\right] \tag{F.7}$$

$$\lambda_{u,t}^O = \beta_u(1+r_t)E_t \lambda_{u,t+1}^O \tag{F.8}$$

F.1.2 Constrained Homeowners and Renters

The constrained homeowners and renters solve the same problems as the homeowners and renters in the two-agent model. The difference from the two-agent model is the proportion of each group in the economy: the proportion of the constrained owners is ω_c and that of the renters is $(1-\omega_u-\omega_c)$. Specifically, the constrained homeowners solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta_c^t \ln X_{c,t}^O \text{ where } X_{c,t}^O \equiv \left[\gamma\left(c_{c,t}^O\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_{c,t}^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.9}$$

$$\text{s.t. } c_{c,t}^O + \rho_t s_{c,t}^O + q_t[h_{c,t} - (1-\delta)h_{c,t-1}] + d_{c,t} = y_t + \frac{d_{c,t+1}}{1+r_t} + \rho_t h_{c,t} \tag{F.10}$$

$$d_{c,t+1} \le m q_t h_{c,t} \tag{F.11}$$

where the superscript "O" represents a homeowner as in the two-agent model, and the subscript "c' a constrained owner. β_c is a subjective discount factor, satisfying $\beta_c < \beta_u$. This condition of discount factor implies the constrained owners are more impatient than the unconstrained owners. In the case of the borrowing constraint, we assume that the constraint is binding near the steady state. This is because the constrained homeowners are sufficiently impatient compared to the unconstrained owners. Formally, I assume $\beta_u(1+r^*) = 1$ and $\beta_c(1+r^*) < 1$, where r^* is the steady-state level of the interest rate.

Given initial values $\{h_{c,-1}, d_{c,0}\}$, the constrained homeowner chooses $\{c_{c,t}^O, s_{c,t}^O, h_{c,t}, d_{c,t+1}\}$ to maximize Equation (F.9) subject to Equations (F.10) and (F.11). The Lagrange multipliers for each constraint are defined as $\lambda_{c,t}^O$ and $\lambda_{c,t}^O \mu_t$, respectively. The corresponding first order conditions are as follows:

$$\gamma\left(X_{c,t}^O\right)^{\frac{1-\eta}{\eta}}\left(c_{c,t}^O\right)^{-\frac{1}{\eta}} = \lambda_{c,t}^O \tag{F.12}$$

$$\frac{1-\gamma}{\gamma}\left(\frac{c^O_{c,t}}{s^O_{c,t}}\right)^{\frac{1}{\eta}}=\rho_t \tag{F.13}$$

$$\lambda^O_{c,t}q_t=\lambda^O_{c,t}\rho_t+\beta_c(1-\delta)E_t\left[\lambda^O_{c,t+1}q_{t+1}\right]+m\mu_t\lambda^O_{c,t}q_t \tag{F.14}$$

$$\lambda^O_{c,t}\left(\frac{1}{1+r_t}-\mu_t\right)=\beta_cE_t\lambda^O_{c,t+1} \tag{F.15}$$

The renter's problem is identical to the two-agent model. As a result, the renter chooses optimal c^R_t and s^R_t at every period to maximize her periodic utility function $\ln X^R_t$ subject to her periodic resource constraint (F.17):

$$\max E_0\sum_{t=0}^{\infty}\beta^t\ln X^R_t \text{ where } X^R_t\equiv\left[\gamma\left(c^R_t\right)^{\frac{\eta-1}{\eta}}+(1-\gamma)\left(s^R_t\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.16}$$

$$\text{s.t. } c^R_t+\rho_ts^R_t=y_t \tag{F.17}$$

and we obtain the first order conditions of Equations (F.18) and (F.19):

$$\gamma\left(X^R_t\right)^{\frac{1-\eta}{\eta}}\left(c^R_t\right)^{-\frac{1}{\eta}}=\lambda^R_t \tag{F.18}$$

$$\frac{1-\gamma}{\gamma}\left(\frac{c^R_t}{s^R_t}\right)^{\frac{1}{\eta}}=\rho_t \tag{F.19}$$

F.1.3 Equilibrium

International financial market is specified as EDEIR given by

$$r_t=r^*+\psi\left(e^{\omega_ud_{u,t+1}+\omega_cd_{c,t+1}-\bar{d}}-1\right) \tag{F.20}$$

where r^* is the steady-state level of the interest rate, ψ is an adjustment parameter and $\bar{d}$ denotes the steady-state level of aggregate debt.

The clearing condition of the housing rental market is to equate the aggregate amount of rental supply to the aggregate amount of rental demand. Since the economy consists of $(\omega_u+\omega_c)$ homeowners and $(1-\omega_u-\omega_c)$ renters, the market clearing condition is

$$\omega_us^O_{u,t}+\omega_cs^O_{c,t}+(1-\omega_u-\omega_c)s^R_t=\omega_uh_{u,t}+\omega_ch_{c,t} \tag{F.21}$$

The supply side of housing rental services is determined by the two types of homeowners' first order conditions (F.8) and (F.15), and the demand side is determined by the renters' FOC (F.19). Hence, we can solve for the rental market equilibrium using Equations (F.8), (F.15), and (F.19) together with the above market clearing condition (F.21). The equilibrium amount of housing rental services and the equilibrium housing rental prices are determined by the following equations:

$$s^O_{c,t}\left(\omega_uc^O_{u,t}+\omega_cc^O_{c,t}\right)=c^O_{c,t}\left((\omega_uh_{u,t}+\omega_ch_{c,t})-(1-\omega_u-\omega_c)s^R_t\right) \tag{F.22}$$

$$\begin{aligned}s^R_t\Big((1-\omega_u-\omega_c)c^R_t\left(\omega_cc^O_{c,t}+(1-\omega_u-\omega_c)c^R_t\right)-\omega_uc^O_{u,t}\left(\omega_uc^O_{u,t}+\omega_cc^O_{c,t}\right)\Big)\\=(\omega_uh_{u,t}+\omega_ch_{c,t})c^R_t\left((1-\omega_u-\omega_c)c^R_t-\omega_uc^O_{u,t}\right)\end{aligned} \tag{F.23}$$

$$\rho_t=\frac{1-\gamma}{\gamma}\left(\frac{c^R_t}{s^R_t}\right)^{\frac{1}{\eta}}\frac{1}{e^{\tau(\rho_t-\bar{\rho})}} \tag{F.24}$$

where $1/e^{\tau(\rho_t-\bar{\rho})}$ in Equation (F.24) represents the rental price adjustment term as we discussed in the two-agent model. Last, we can construct the equilibrium process of aggregate variables by

$$HC_t=\rho_t(\omega_uh_{u,t}+\omega_ch_{c,t}) \tag{F.25}$$

$$NHC_t=\omega_uc^O_{u,t}+\omega_cc^O_{c,t}+(1-\omega_u-\omega_c)c^R_t \tag{F.26}$$

$$TC_t=HC_t+NHC_t \tag{F.27}$$

$$H_t=\omega_uh_{u,t}+\omega_ch_{c,t} \tag{F.28}$$

$$D_{t+1} = \omega_u d_{u,t+1} + \omega_c d_{c,t+1} \tag{F.29}$$

$$Y_t = y_t + \rho_t(\omega_u h_{u,t} + \omega_c h_{c,t}) \tag{F.30}$$

F.2 Full Equilibrium Conditions

An equilibrium in the model is a set of processes of $\{c^O_{u,t}, c^O_{c,t}, c^R_t, h_{u,t}, h_{c,t}, s^O_{u,t}, s^O_{c,t}, s^R_t, d_{u,t+1}, d_{c,t+1}, X^O_{u,t}, X^O_{c,t}, X^R_t, \lambda^O_{u,t}, \lambda^O_{c,t}, \lambda^R_t, \mu_t, \rho_t, r_t, TC_t, HC_t, NHC_t, H_t, D_{t+1}, Y_t\}_{t=0}^{\infty}$ satisfying (F.1) to (F.30) given the processes $\{q_t, y_t\}_{t=0}^{\infty}$ and the initial condition $\{h_{u,-1}, h_{c,-1}, d_{u,0}, d_{c,0}\}$. Therefore, there are 25 endogenous variables, and we need 25 equations to determine equilibrium. Also, we need 2 equations which specify exogenous variable processes.

$$\gamma\left(X^O_{u,t}\right)^{\frac{1-\eta}{\eta}}\left(c^O_{u,t}\right)^{-\frac{1}{\eta}} = \lambda^O_{u,t} \tag{F.31}$$

$$\gamma\left(X^O_{c,t}\right)^{\frac{1-\eta}{\eta}}\left(c^O_{c,t}\right)^{-\frac{1}{\eta}} = \lambda^O_{c,t} \tag{F.32}$$

$$\gamma\left(X^R_t\right)^{\frac{1-\eta}{\eta}}(c^R_t)^{-\frac{1}{\eta}} = \lambda^R_t \tag{F.33}$$

$$\rho_t = \frac{1-\gamma}{\gamma}\left(\frac{c^R_t}{s^R_t}\right)^{\frac{1}{\eta}}\frac{1}{e^{\tau(\rho_t-\bar{\rho})}} \tag{F.34}$$

$$s^O_{c,t}\left(\omega_u c^O_{u,t} + \omega_c c^O_{c,t}\right) = c^O_{c,t}\left((\omega_u h_{u,t} + \omega_c h_{c,t}) - (1-\omega_u-\omega_c)s^R_t\right) \tag{F.35}$$

$$\begin{aligned} & s^R_t\left((1-\omega_u-\omega_c)c^R_t\left(\omega_c c^O_{c,t} + (1-\omega_u-\omega_c)c^R_t\right) - \omega_u c^O_{u,t}\left(\omega_u c^O_{u,t} + \omega_c c^O_{c,t}\right)\right) \\ & \quad = (\omega_u h_{u,t} + \omega_c h_{c,t})c^R_t\left((1-\omega_u-\omega_c)c^R_t - \omega_u c^O_{u,t}\right) \end{aligned} \tag{F.36}$$

$$\omega_u s^O_{u,t} + \omega_c s^O_{c,t} + (1-\omega_u-\omega_c)s^R_t = \omega_u h_{u,t} + \omega_c h_{c,t} \tag{F.37}$$

$$\lambda^O_{u,t} = \beta_u(1+r_t)E_t\lambda^O_{u,t+1} \tag{F.38}$$

$$\lambda^O_{c,t}\left(\frac{1}{1+r_t} - \mu_t\right) = \beta_c E_t \lambda^O_{c,t+1} \tag{F.39}$$

$$\lambda^O_{u,t}q_t = \lambda^O_{u,t}\rho_t + \beta_u(1-\delta)E_t\left[\lambda^O_{u,t+1}q_{t+1}\right] \tag{F.40}$$

$$\lambda^O_{c,t}q_t = \lambda^O_{c,t}\rho_t + \beta_c(1-\delta)E_t\left[\lambda^O_{c,t+1}q_{t+1}\right] + m\mu_t\lambda^O_{c,t}q_t \tag{F.41}$$

$$X^O_{u,t} \equiv \left[\gamma\left(c^O_{u,t}\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s^O_{u,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.42}$$

$$X^O_{c,t} \equiv \left[\gamma\left(c^O_{c,t}\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s^O_{c,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.43}$$

$$X^R_t \equiv \left[\gamma(c^R_t)^{\frac{\eta-1}{\eta}} + (1-\gamma)(s^R_t)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{F.44}$$

$$c^O_{u,t} + \rho_t s^O_{u,t} + q_t[h_{u,t} - (1-\delta)h_{u,t-1}] + d_{u,t} = y_t + \frac{d_{u,t+1}}{1+r_t} + \rho_t h_{u,t} \tag{F.45}$$

$$c^O_{c,t} + \rho_t s^O_{c,t} + q_t[h_{c,t} - (1-\delta)h_{c,t-1}] + d_{c,t} = y_t + \frac{d_{c,t+1}}{1+r_t} + \rho_t h_{c,t} \tag{F.46}$$

$$d_{c,t+1} = mq_t h_{c,t} \tag{F.47}$$

$$c_t^R + \rho_t s_t^R = y_t \tag{F.48}$$

$$r_t = r^* + \psi\left(e^{\omega_u d_{u,t+1} + \omega_c d_{c,t+1} - \bar{d}} - 1\right) \tag{F.49}$$

$$HC_t = \rho_t(\omega_u h_{u,t} + \omega_c h_{c,t}) \tag{F.50}$$

$$NHC_t = \omega_u c_{u,t}^O + \omega_c c_{c,t}^O + (1 - \omega_u - \omega_c)c_t^R \tag{F.51}$$

$$TC_t = HC_t + NHC_t \tag{F.52}$$

$$H_t = \omega_u h_{u,t} + \omega_c h_{c,t} \tag{F.53}$$

$$D_{t+1} = \omega_u d_{u,t+1} + \omega_c d_{c,t+1} \tag{F.54}$$

$$Y_t = y_t + \rho_t(\omega_u h_{u,t} + \omega_c h_{c,t}) \tag{F.55}$$

$$\ln q_{t+1} = \phi_q \ln q_t + \sigma_q \varepsilon_{q,t+1} \tag{F.56}$$

$$\ln y_{t+1} = \phi_y \ln y_t + \sigma_y \varepsilon_{y,t+1} \tag{F.57}$$

F.3 The Steady States of the Economy

First of all, r^* is the steady state level of interest rate, by definition. From (F.56) and (F.57), $q^* = 1$ and $y^* = 1$. From (F.39), we can figure out the steady state level of the Lagrange multiplier for the constrained owner's borrowing constraint:

$$\mu^* = \frac{1}{1 + r^*} - \beta_c$$

As we know μ^*, we can earn the steady state level of rental price, ρ^*, from (F.41):

$$\rho^* = 1 - \beta_c(1 - \delta) - m\mu^*$$

Moving on to (F.34), s_R^* can be re-written by

$$s_R^* = \left(\frac{\gamma}{1-\gamma}\right)^{-\eta} \rho^{*-\eta} c_R^* \tag{F.i}$$

By plugging (F.i) into the renter's budget constraint (F.48), we get the steady state level of renter's non-housing consumption, c_R^*:

$$c_R^* = \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}} \tag{F.ii}$$

Moving on the unconstrained homeowner's resource constraint (F.45), it is evaluated at the steady states and expressed as:

$$c_{O,u}^* + \rho^* s_{O,u}^* + (\delta - \rho^*)h_u^* + \frac{r^*}{1 + r^*} d_u^* = 1$$

Since we can express $s_{O,u}^*$ in the function of $c_{O,u}^*$ using the first order condition of the unconstrained owner's problem, the above equation is an equation in terms of the undetermined $c_{O,u}^*$, h_u^* and d_u^*:

$$\left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}\right) c_{O,u}^* + (\delta - \rho^*)h_u^* + \frac{r^*}{1 + r^*} d_u^* - 1 = 0 \tag{F.iii}$$

Likewise, we can obtain the following equation from the constrained homeowner's resource constraint:

$$\left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}\right) c_{O,c}^* + \left(\delta - \rho^* + \frac{mr^*}{1 + r^*}\right) h_c^* - 1 = 0 \tag{F.iv}$$

For the above equation (F.iv), we used $d_c^* = mh_c^*$ which comes from (F.47). Therefore, (F.iv) is an equation in terms of the undetermined $c_{O,c}^*$ and h_c^*. We set the steady state level of debts of two types of homeowners are equal, which means:

$$d_u^* = mh_c^* \tag{F.v}$$

From (F.36) and (F.37), we can earn two more equations of $c_{O,u}^*$, $c_{O,c}^*$, h_u^*, h_c^* and d_u^*:

$$\begin{aligned} & s_R^*\Big((1-\omega_u-\omega_c)c_R^*\Big(\omega_c c_{O,c}^* + (1-\omega_u-\omega_c)c_R^*\Big) - \omega_u c_{O,u}^*\Big(\omega_u c_{O,u}^* + \omega_c c_{O,c}^*\Big)\Big) \\ & \quad = (\omega_u h_u^* + \omega_c h_c^*)c_R^*\Big((1-\omega_u-\omega_c)c_R^* - \omega_u c_{O,u}^*\Big) \end{aligned} \tag{F.vi}$$

$$\left(\frac{\gamma}{1-\gamma}\right)^{-\eta} \rho^{*-\eta}\left(\omega_u c_{O,u}^* + \omega_c c_{O,c}^*\right) + (1-\omega_u-\omega_c)s_R^* = \omega_u h_u^* + \omega_c h_c^* \tag{F.vii}$$

Note that, in (F.vi) and (F.vii), s_R^* and c_R^* are given by (F.i) and (F.ii), respectively. As a result, for the undetermined $c_{O,u}^*$, $c_{O,c}^*$, h_u^*, h_c^* and d_u^*, we can solve the system of equations comprised of (F.iii) to (F.ii). When we obtain the fixed points for $c_{O,u}^*$, $c_{O,c}^*$, h_u^*, h_c^* and d_u^*, we can figure out the other remaining endogenous variables' steady-state levels according to the equilibrium conditions.

Appendix G. The Two-Agent Model: Emerging vs. Rich Economy

As described in the main text, I hypothesize that homeowners in developed countries face looser credit conditions and thus a lower probability of hitting the borrowing constraint. However, there is a challenge in controlling the hitting frequency in the two-agent model with occasionally binding borrowing constraint. For this reason, the counterfactual model corresponding to the two-agent model assumes no collateral constraint. Therefore, this appendix presents the comparison between the model with occasionally binding constraint for emerging economies and the model without borrowing constraint for rich economies, as an alternative. Table G1 presents the parameter values for the counterfactual economy which

Table G1
Calibration for Rich Economy (Canada).

Parameter		Value
Share of non-housing consumption	γ	0.53
Intratemporal elasticity of substitution	η	0.74
Steady state of interest rate	r^*	0.01
Subjective discount factor	β	0.99
Housing depreciation rate	δ	0.0025
Loan-to-value ratio	m	0.9
Homeowner proportion	ω	0.69
Interest rate premium adjustment	ψ	0.000742
Rental price adjustment	τ	2,250
Persistence of house prices	ϕ_q	0.8463
Persistence of endowment	ϕ_y	0.8853
Standard deviation of innovation to house price shock	σ_q	0.0192
Standard deviation of innovation to endowment shock	σ_y	0.0066

Table G2
Selected Second Moments: Emerging vs. Rich.

	Korea[1]	Emerging[2]	Canada[1]	Rich[2]
σ_C/σ_Y	1.19	1.20	0.74	0.80
σ_{HC}/σ_Y	1.64	1.78	0.76	4.58
σ_{NHC}/σ_Y	1.39	1.17	0.90	0.46
σ_Y	2.41	2.61	1.42	7.12
σ_q	5.16	5.16	3.52	3.52
σ_ρ	1.89	1.88	1.58	1.58
$\rho_{C,Y}$	0.70	0.59	0.49	0.39
$\rho_{HP,Y}$	0.57	0.39	0.35	0.34
$\rho_{HP,C}$	0.38	0.42	0.22	0.17
$\rho_{HP,HC}$	0.03	0.46	−0.22	0.58
$\rho_{HP,NHC}$	0.38	0.21	0.27	0.13

[1] Seasonally-adjusted HP-filtered series ($\lambda = 1,600$), 1975 Q1-2016 Q3.
[2] Two-agent model with occasionally binding borrowing constraint, solved using a piecewise linear approximation method in Guerrieri and Iacoviello (2015): "Emerging" uses the parameter values in Table 3, and "Rich" uses those in Table G1.

is Canada. All values are identical to the Table 7 except β, ω, and τ. The subjective discount factor β is set to 0.99 to satisfy the condition of $\beta(1+r^*)=1$ which is necessary for the model without the borrowing constraint. The homeownership rate parameter ω is 0.69, and the rental price adjustment parameter τ is set to match the ratio of rent volatility to house price volatility in Canada, which is 0.39.

The first two columns of Table G2 come from Table 5 in Section 4. The first column shows the quarterly business cycle moments in South Korea, and the second column shows the moments yielded by the baseline two-agent model under the baseline calibration. As we discussed, the baseline model captures the actual data well. In contrast, the model for the counterfactual economy without a collateral constraint (the forth column under the heading "Rich") does not match some moments observed in the Canadian economy (the third column under the heading "Canada"). While the approximation of the counterfactual economy generates non-excessive total consumption as we observe in the Canadian data (0.74 and 0.80, respectively), there are huge consumption and income fluctuations in the theoretical world, as $\sigma_Y = 7.12$ indicates. Moreover, there is much greater volatility in housing consumption, even though rental price volatility is suppressed by a rent adjustment mechanism. The main reason behind these results is that the housing stock or the amount of housing services is significantly volatile in the economy without a borrowing constraint. Since homeowners can buy and sell housing as much as they want given exogenous housing prices, the volatility of housing stock becomes excessive. Note that the relative non-housing consumption volatility is quite low, $\sigma_{NHC}/\sigma_Y = 0.46$. This is because the rental price or the relative price of non-housing consumption, ρ, is sticky near the steady state value. Finally, it is notable that the correlation coefficients between the counterfactual economy ("Rich") and the baseline economy ("Emerging") support the evidence of correlations suggested in Section 2. That is, consumption is more correlated with house prices in the baseline case ($\rho_{HP,C} = 0.42$) than in the counterfactual case ($\rho_{HP,C} = 0.17$), while the correlation between output and house prices for the baseline case ($\rho_{HP,Y} = 0.39$) countries is at a similar level as that of the counterfactual case ($\rho_{HP,Y} = 0.34$). However, in overall performance, "Rich" fails to explain Canadian business cycle facts though it matches some moments of interest. In fact, this counterfactual economy is an extreme variation of the baseline two-agent model in that it completely wipes a collateral constraint out.

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