Midterm Exam

(October 20, 2023, **1 hour 15 minutes**)

Macroeconomics (Fall 2023)

Professor: Wonmun Shin

** Write up your answers as **clearly**, **precisely**, and **concisely** as possible. Your grade will be reduced if your answer is unreasonably difficult to follow.*
** Label the axes and curves when you draw graphs.*

1. **(Total 50 points)** Assume that an economy has the production function:

$$Y = AK^{\frac{1}{3}}L^{\frac{1}{3}}G^{\frac{1}{3}}$$

where K is physical capital, L is labor (which we assume to be equal to population), A is the level of technology, and G is the stock of government infrastructure (which we assume to be rival goods, *eg.* highways, airports).

(a) **(5 points)** Does the above production function exhibit constant returns to scale (CRTS)? Show.

(b) **(5 points)** Define output per capita as $y = Y/L$, capital per capita as $k = K/L$, and infrastructure per capita as $g = G/L$. Write down the production function in ***per-capita*** terms.

(c) **(5 points)** Does the production function in per-capita terms you derived in part (b) exhibit diminishing returns to k (capital per capita)? Show.

Imagine that the economy is closed, and that the saving rate is the constant fraction s and the depreciation rate is δ. Population growth is exogenous constant number n.

(d) **(5 points)** Write down the fundamental equation of the Solow-Swan model for this economy. That is, write down the equation for dk/k as a function of the parameters A, s, n, δ, the capital stock k, and the level of infrastructure per capita g.

(e) **(5 points)** According to the FESS derived in part (d), draw the savings curve and depreciation line on the plane whose horizontal axis is k. Is there a steady state? Is it unique?

(f) **(5 points)** Consider the parameters $A = 1$, $s = 0.2$, $\delta = 0.09$, $n = 0.01$. Suppose that the amount of infrastructure per person is constant $g = 8$. What is the steady state capital stock per capita, k^*, in this economy?

Imagine that the economy is stuck at the steady state with no growth. Suppose that the World Bank gives aid to this country. We are going to analyze consequences of this aid. Notice that the parameter values for A, s, δ, and n are same as in the part (f).

(g) **(10 points)** Imagine that the aid takes the form of physical capital k (for example, machinery). Specifically, the World Bank provides 19 units of k to the country, so the economy has a stock of capital per capita equal to $k^* + 19$ after the aid. Note that g remains constant at $g = 8$. How does the economy behave (i) **IMMEDIATELY** after the aid, and (ii) in the **LONG TERM**? (iii) Calculate the **STEADY STATE CAPITAL PER CAPITA** of the economy after the aid.

(h) **(10 points)** Imagine instead that aid takes the form of infrastructure with the same amount. (Note: Forget about the situation in part (g).) That is, imagine that g increases from $g = 8$ to $g = 27$. How does the economy behave (i) **IMMEDIATELY** after the aid, and (ii) in the **LONG TERM**? (iii) Calculate the **STEADY STATE CAPITAL PER CAPITA** of the economy after the aid.

2. **(Total 50 points)** Consider a consumer who lives for only two periods. She receives incomes Y_1 and Y_2 in period 1 and period 2 respectively. The consumer has access to a financial market where she transacts a financial instrument, B_t, at the interest rate r. Assume that the consumer is born with no assets or debts ($B_0 = 0$) and dies with no bonds ($B_2 = 0$). She wants to choose the amount of consumption in period 1 and 2 (C_1 and C_2, respectively) that maximizes her utility subject to a budget constraint.

(a) **(5 points)** Write the dynamic budget constraint (DBC) the consumer faces. Then, specify the budget constraint for period 1 and the budget constraint for period 2.

(b) **(5 points)** Derive the intertemporal budget constraint (IBC) based on the budget constraints you obtained in part (a).

(c) **(5 points)** Draw the IBC you obtained in part (b) in a diagram that has C_1 and C_2 in the axes. (Note: Don't forget to mark the endowment point (Y_1, Y_2).) What is the slope of the budget constraint?

(d) **(5 points)** Suppose that the consumer likes to ***smooth consumption over periods***. Display her optimal choice, where she becomes a saver (or a lender) at period 1.

(e) **(5 points)** Imagine that there is a government who imposes **TAX** on the consumer ***only in period 1***. In other words, the consumer should pay T_1 in period 1, and $0 < T_1 < Y_1$. How does her consumption in period 1 change? Explain using a graph.

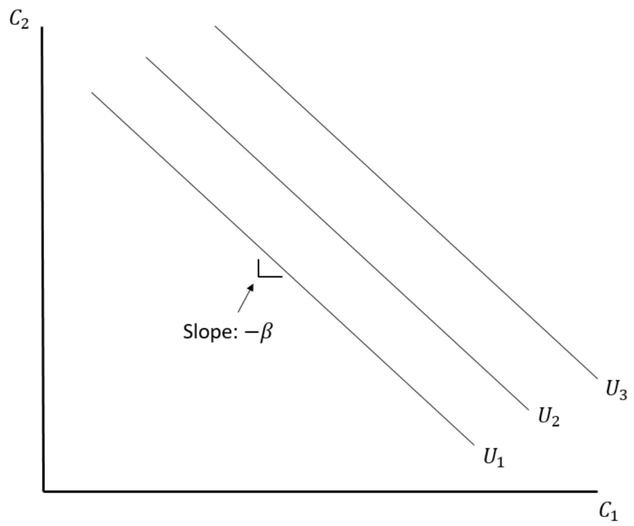
(f) **(5 points)** Imagine instead that, in period 1, the government announces that it will provide the consumer with **SUBSIDY** ***in period 2***. (Note: Forget about the situation in part (e).) Specifically, the government will pay S_2 to the consumer in period 2, and $S_2 > 0$. Does this announcement affect her consumption in period 1, or not? Explain using a graph.

(g) **(5 points)** Imagine instead that, in period 1, the government issues a **ONE-PERIOD GOVERNMENT BOND** (G) with the interest rate r and the consumer must buy the bond. (Note: Forget about the situation in parts (e) and (f).) As a result, the consumer should pay G in period 1, and she will earn G and the interest rG in period 2. Does her consumption in period 1 change after the government bond issuance, or not? Explain using equations or a graph.

From now, let us consider that the consumer's preference is linear. (Note: Forget about the situation in parts (d), (e), (f), and (g).) To be specific, her utility function is given by:

$$U(C_1, C_2) = C_1 + \frac{1}{\beta} C_2$$

where $1/\beta$ is a discount factor for future consumption, and $\beta > 1$. The set of indifference curves look like:



where U_2 is greater than U_1, and U_3 is greater than U_2. Notice that the slope of linear indifference curve is β.

(h) **(5 points)** Consider that $Y_1 = 100$, $Y_2 = 110$, and $r = 0.1$. Draw the IBC on the plane whose x-axis is C_1 and y-axis is C_2. (Note: Do not forget to mark the endowment point.) What are the values for x-intercept and y-intercept?

(i) **(5 points)** Consider $\beta = 1.2$. What is the consumer's optimal consumption in period 1? Is she a borrower or a saver? (Note: The optimal level of C_1 must be expressed in numerical value.)

(j) **(5 points)** Consider still $\beta = 1.2$ but the interest rate changes to $r = 0.3$. What is the consumer's optimal consumption in period 1? Is she a borrower or a saver? (Note: The optimal level of C_1 must be expressed in numerical value.)

(End of Exam, Total 3 Pages)