Midterm Exam

(April 26, 2021, **1 hour 15 minutes**)

Econometrics (Spring 2021)

Professor: Wonmun Shin

Part I

True or False

(Total 30 points) Read each statement below carefully. Write a **T** if you think the statement is True. Write an **F** if you think the statement is false. Do not forget to state a reason precisely if you answer F. (Grading guide: If your answer (with a reason in case F) is correct, you will get 5 points. If your answer is partially correct (*i.e.* F without a reason, F with an incorrect reason, etc.), you will get partial points.)

1. In order to prove Gauss-Markov Theorem, normality of regression error should be assumed.

2. The higher is the value of σ^2 (variance of regression error e_i), the larger is the variance of OLS estimator $\hat{\beta}_2$ in the simple regression.

3. Even though the regression errors are not normally distributed, OLS estimators for intercept and slope coefficients are still unbiased.

4. The p-value and the size of a test in the hypothesis testing problems are the same.

5. The more observation you have, you will get more accurate estimates. Therefore, R^2 will always be higher if you have more observations.

6. Since $E\left(\hat{\beta}_2\right) = \beta_2$ in the simple regression, the OLS estimator $\hat{\beta}_2$ is consistent when sample size gets large.

Part II

Short Questions

1. **(10 points)** Consider a two-variable simple regression, *i.e.* $Y_i = \beta_1 + \beta_2 X_i + e_i$. *Student A* thinks that a slope parameter β_2 is more important and an intercept is not necessary, so she estimates $Y_i = \beta_2^A X_i + e_i$ without an intercept though the true model is $Y_i = \beta_1 + \beta_2 X_i + e_i$. In case, her OLS estimator $\hat{\beta}_2^A$ is an unbiased estimator of the true parameter β_2? (Yes or No without any explanation will be 0 points.)

2. **(10 points)** Suppose *Student B* runs the following regression:

$$y_i = \alpha_1 + \alpha_2 x_i + u_i$$

where, as usual, y_i and x_i are deviations from their respective mean values, *i.e.* $y_i = \left(Y_i - \bar{Y}\right)$, $x_i = \left(X_i - \bar{X}\right)$. What will be the OLS estimators $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in the *Student B*'s regression? Are they different from the OLS estimators in the regression $Y_i = \beta_1 + \beta_2 X_i + e_i$?

3. **(10 points)** *Student C* surveyed 50 classmates to examine the relationship between exam scores of *Professor Shin*'s Econometrics (Y) and studying hours (X). After the survey, he obtained the following results:

$$\sum X_i = 450, \quad \sum Y_i = 2,400$$

$$\sum \left(X_i - \bar{X}\right)^2 = 800 \quad \sum \left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) = 3,200$$

Suppose that *Student C* wants to obtain the OLS estimators for a simple regression model, $Y_i = \beta_1 + \beta_2 X_i + e_i$. Using the above results, calculate $\hat{\beta}_1$ and $\hat{\beta}_2$.

Part III

Long Questions

1. **(Total 20 points)** Consider the following regression models:

$$\textbf{Model 1}: Y_i = \beta_1 + \beta_2 X_i + e_i$$
$$\textbf{Model 2}: Y_i^* = \beta_1^* + \beta_2^* X_i^* + e_i^*$$

where $Y_i^* = 2Y_i$ and $X_i^* = \frac{1}{2}X_i$. Assume that the classical assumptions are satisfied .

(a) **(4 points)** Find the OLS estimators of β_1 and β_2 in **Model 1**.

(b) **(4 points)** Find the OLS estimators of β_1^* and β_2^* in **Model 2**. Are they identical to $\hat{\beta}_1$ and $\hat{\beta}_2$ you obtained in (a)?

(c) **(2 points)** Is the R^2 different between **Model 1** and **Model 2**?

Student D considers the log-linear models by taking log on both dependent variable and regressor in **Model 1** and **Model 2**:

$$\textbf{Model 3}: \ln Y_i = \alpha_1 + \alpha_2 \ln X_i + u_i$$
$$\textbf{Model 4}: \ln Y_i^* = \alpha_1^* + \alpha_2^* \ln X_i^* + u_i^*$$

(d) **(4 points)** Discuss the difference between the OLS estimator of β_2 in **Model 1** and that of α_2 in **Model 3**, in terms of interpretation.

(e) **(4 points)** Establish the relationship between $\hat{\alpha}_2$ from **Model 3** and $\hat{\alpha}_2^*$ from **Model 4**. Are they identical?

(f) **(2 points)** Is the R^2 different between **Model 3** and **Model 4**?

2. **(Total 20 points)** Consider the following regression output:

$$\widehat{\text{Income}} = \underset{(8541)}{26191} + \underset{(720)}{5040}\,\text{Education}$$
$$n = 251 \quad RSS = 50000 \quad ESS = 30000$$
$$\text{(standard errors in parentheses)}$$

where *Income* is annual income in dollars, *Education* is educational attainment in years.

(a) **(5 points)** How do you interpret this regression?

(b) **(5 points)** Test the hypothesis: $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 \neq 0$ at 5% significance level. (Note: $t_{\alpha/2}$ is 1.64 when $\alpha = 0.1$, 1.96 when $\alpha = 0.05$, and 2.57 when $\alpha = 0.01$.)

(c) **(5 points)** Report R^2 of this regression.

(d) **(5 points)** Calculate the OLS estimator of σ^2 using the above regression output.